Elastic Waves in Random Media. Fundamentals of Seismic Stratigraphic Filtering

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ABSTRACT

In August 1998 we finished our book with the title above. This book treats various generalizations of the classical O'Doherty-Anstey formula in order to describe stratigraphic filtering effects. These are the effects that can be observed when acoustic, elastic and electromagnetic waves propagate through a stack of thin layers. Below we give a short description of the contents of the book.

SUMMARY OF THE BOOK

Stacks of thin layers are a physical reality in any sedimentary basin. Typical sonic logs (as well as shear-wave and density logs) show that sedimentary rocks are composed of layers with thicknesses of the order of $0.1m$ to $10m$. Sometimes a micro layering of even smaller characteristic sizes can be observed. This means that to a reservoir at $4km$ depth the wave field must transmit through hundreds or thousands of layers. *The influence of thinly layered structures on transmitted seismic wavefields is called stratigraphic filtering.* An understanding and a reliable description of this effect is of importance for seismic exploration and seismology. Of course, there never exists a perfectly parallel layered geological structure. However, in many cases this model, also called 1-D model (note that all media under consideration are assumed to be heterogeneous along one spatial axis only) can be used as a first and useful approximation to describe the stratigraphic filtering.

On the other hand, multilayered stacks can be observed not only in the solid Earth. Oceans, atmosphere, artificial composite materials and some biological media very often have similar structures. Moreover, many real systems are 1-D or quasi 1-D just due to their physical nature (like e.g., communication lines). Thus, the problems of stratigraphic filtering are of more general character than seismological only.

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The attention of seismologists and exploration geophysicists has long been attracted to the problem of seismic wave propagation in parallel layered 1-D elastic media. There exists a number of books, which are entirely or partly devoted to seismic waves in layered structures (Aki and Richards, 1980; Robinson and Treitel, 1980; Brekhovskikh and Godin, 1989; Kennett, 1983; Tygel and Hubral, 1987, etc.), without however, giving attention to the diverse aspects of stratigraphic filtering as done here.

Often an exact computation of the recorded wavefield (i.e., the synthetic seismogram) in multilayered media is unwieldy and in fact not necessary. Moreover, the exact information on the physical parameters of these layers is never known. The principal task of this book is to provide a simple description of mean properties (like the attenuation coefficient and the phase velocity) of the transmitted wavefield in terms of a very restricted number of statistical properties of the medium (e.g., the correlation distance and the variances of the density and velocity-log fluctuations). These statistics can be expected to be more reliable than the details in the log measurements, which are always influenced by the drilling- and logging techniques.

The so-called generalized O' Doherty-Anstey formulas, which are a central topic in this book, fulfill the above mentioned task. In contrast to deterministic models and approaches discussed in the above mentioned books, here the seismic wave propagation in sediments is considered as a problem of wave propagation in randomly multilayered 1-D media. Effects of non perfect 1-D multilayering are not considered. The attention is mainly concentrated on properties of plane wavefields normally or obliquely transmitted through a multilayered 1-D medium (i.e., transmissivity). An approximation of wavefields being reflected from such a medium (i.e., reflectivity) is also discussed. Effects of inelasticity are touched in the considerations of poroelastic media and AVO-corrections.

The book is organized as follows:

In Chapter 1 we give a short description of the main assumptions, concepts and results of this book and a historical introduction to the subject.

In Chapter 2 we review the basic and most important concepts of the theory of random processes as well as some important statistical and physical terms related to localization, random media, averaging and self-averaged quantities.

In Chapter 3 we discuss normal-incidence plane-wave propagation in the most simple and in seismic processing widely used 1-D model of sediments, i.e. the Goupillaud model (Goupillaud, 1961) - where the stratification is represented by homogeneous layers equidistantly spaced in traveltime. A plane wavefield propagates in such systems as a single mode. In spite of its very schematic character the Goupillaud model is of great practical importance. Important signal processing steps, like e.g., deconvolution have been designed for this model. It provides a good possibility to understand the behavior of not only the main part of the transmitted wavefield following immediately its first arrival, but its coda also. Additionally, a very clear description of the

reflectivity is possible in the frame of this model.

In Chapter 4 we continue to consider wavefield propagation in the single-mode regime. Now, however, the oblique incidence in general 1-D inhomogeneous structures is under consideration. We obtain the attenuation coefficients and vertical phase increments (i.e., real parts of vertical components of the wave vectors) for transmissivities in the case of scalar waves. These can be pressure waves in a fluid or SH-polarized shear waves in a solid. The corresponding derivation is performed in a small-perturbation approximation. This is valid up to the second order in the fluctuations of the density and elastic modulus, i.e., the validity domain of the O' Doherty-Anstey approximation. One can view the result as being a second-order Rytov approximation.

In Chapter 5 we consider multi-mode wavefields. The case of compression- (P) or shear- (SV) waves (i.e., vector wavefields) obliquely propagating in a 1-D heterogeneous solid will be studied. Relatively simple explicit formulas are obtained for the attenuation coefficients and phase increments of the transmitted waves. Finally, in this chapter, an approach is presented which is based on the invariant-embedding method. This provides an elegant and rather general way of solution for the reflected as well as transmitted wavefields.

In Chapter 6 the solutions obtained for scalar and vector wavefields in elastic media are used to discuss the seismic time-harmonic transmissivity in more depth. Its kinematic and dynamic properties like phase velocities and attenuation coefficients are analyzed. Such effects like frequency-dependent anisotropy, shear-wave splitting and angle-dependent attenuation are described.

In Chapter 7 these results are used to derive the transmissivity in the time domain (i.e., the transient transmissivity). In this chapter we also give a number of numerical illustrations to better understand the theory.

In Chapter 8 we discuss practical applications of the theory of stratigraphic filtering for amplitude corrections in the amplitude-variation with offset (AVO) analysis.

In Chapter 9 we consider the multi-mode propagation of the wavefield in poroelastic 1-D inhomogeneous media. We describe there an important mechanism of the attenuation of seismic waves called the inter-layer flow. In addition, we approach the question of seismic signatures of the permeability.

In Chapter 10 a short discussion of the reflectivity of elastic waves finalizes our consideration.

In this book, which is dedicated to a very specialized topic of seismic-wave-propagation theory, we must assume that the reader possesses certain backgrounds of wave propagation, statistics, communication theory, etc. Depending on the individual background, she or he would find access to the whole book by not necessarily starting with Chapter 1. One can start selecting the chapters with the subject one is most familiar with and proceed from these to the less familiar topics in the other chapters.

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4. This book treats various generalizations of the classical O'Doherty-Anstey formula in order to describe stratigraphic filtering effects. These are the effects that can be observed when elastic and electromagnetic waves propagate through multilayered structures. Our aim was to treat this topic in a comprehensive manner and present compact results in a didactically simple way, emphasizing the physics of the wave-propagation phenomena. We do not claim mathematical rigidity in all our derivations, however, we are pleased to have obtained quite simple descriptions of scattering, transmissio