

Linear Programming-Based Optimization of Synthetic Fertilizers Formulation

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Abstract

Linear least-cost programming was used in formulating three NPK labeled synthetic fertilizers. Linear Programming technique was selected to formulate the appropriate composition of the three synthetic fertilizers mixes with least costs of production and optimum potential to increase yield of crops and soil fertility. Data about fertilizers specifications and constraints imposed on the fertilizers components were collected from 30 synthetic fertilizers plants. Costs of ingredients used in fertilizers formulation were obtained from the prevailing market prices. The results of the study prevailed that the least cost synthetic fertilizer combinations of the three studied fertilizer mixes among many calculated combinations was 672 JDs for the first mix with NPK label of 20-20-20, 669 JDs for the second mix with NPK label of 15-30-15, and 754 JDs for the third mix with NPK label of 12-12-36. These all three costs are lower by nearly 5-10% than those imposed by the market. These results confirm the importance of using techniques such as LP to formulate least cost products in industries such as synthetic fertilizers industry.

Keywords: linear programming, synthetic fertilizers, optimization, least cost

1. Introduction

There is a need to rebuild soil fertility to preserve agricultural sustainability and to maintain acceptable levels of productivity and farm income (Heerink et al., 2001). Positive gains in farm productivity as well as improved soil fertility could be achieved through increased levels of fertilizer use (International Center for Soil Fertility and Agricultural Development or IFDC, 2007).

Any organic or inorganic material added to the soil to supply sufficient amounts of elements essential to the growth of plants is a fertilizer. Fertilizers may be categorized into two groups, natural and synthetic fertilizers. Synthetic fertilizers include different types according to their chemical composition, physical status and solubility in water. With application rates depending on the soil fertility synthetic fertilizers are commonly used for growing all crops, (Lowrison, 1989). Lack of labor and time for collection and a ready and easy access to synthetic fertilizers are the main reasons for farmers to use synthetic fertilizers (Thai et al., 2007).

Synthetic fertilizer makes up the majority of nutrient inputs necessary to sustain current crop yields in the USA (Stewart et al., 2005). Synthetic fertilizers have been developed to supply nitrogen (N), phosphorus (P), and potassium (K) in significant amounts. During the last 50 years, the use of synthetic fertilizers has increased steadily. About one-third of the food produced now could not be produced without synthetic fertilizers (Glass, 2003).

A chemical compound (or blend) containing significant quantities of N, P, and K is a complete fertilizer (Aldrich et al., 1986). The total percentage of the nutrients contained in a fertilizer is given as three numbers, which together is known as the analysis. These numbers are usually in large print on the front of the container or bag. An example would be 10-30-10 (Tisdale et al., 1985).

Picking blends of raw materials in synthetic fertilizers formulation to produce finished fertilizer combinations at minimum cost is a very important issue in synthetic fertilizers industry. Formulation of a least-cost balanced synthetic fertilizers mix would offer an opportunity of reducing overall production cost as well as increasing production.

Many quantitative mathematical analysis tools have been developed to analyze and support decision making in agricultural research and farming systems (Agrawal & Heady, 1972). Linear Programming (LP) is the most recognized one. There are hundreds of applications of LP in agriculture (Taha, 1987). For example, LP is one of the most important techniques to allocate the available feedstuffs in a least cost broiler ration formulation (Aletor, 1986; Ali & Leeson, 1995).

The application of LP in least-cost synthetic fertilizer formulation in synthetic fertilizers formulation has gained wide recognition LP is widely used in this area because synthetic agricultural fertilizers companies are interested in minimizing production costs while meeting certain specified levels of nitrogen, phosphate, and potash by blending together a number of raw materials.

For this study, LP was selected to formulate the appropriate composition of synthetic fertilizers mix with least costs of production and optimum potential to increase yield of crops and soil fertility.

2. Linear Programming

Linear programming (LP) is a widely used technique designed to help operations managers plan and make the decisions necessary to allocate resources. It is an optimization process of a linear objective function, subject to linear equality and linear inequality constraints (Kuester & Mize, 1973).

It is a mathematical systems analysis technique, which optimizes some linear objective functions subject to certain linear constraints in order to explore the optimum solution. In agriculture, LP approach is being used for crop rotation, mix of cash crops and food crops and fertilizer mix. LP reflects, in the form of a model, the organization's attempt to mainly maximize profit or minimize costs in view of limited resources.

The term linear describes the proportionate relationship of two or more variables. Thus a given change in one variable will always cause a resulting proportional change in another variable. Linear Programming assumes; (Render et al., 2006)

- 1) Certainty: Certainty means that all the model parameters such as availability of resources, profit (or cost) contribution of a unit of decision variable and consumption of resources by a unit of decision variable must be known and constant.
- 2) Divisibility (Continuity): Divisibility means that the solution values of decision variables and resources are assumed to have either whole numbers (integers) or mixed numbers (integer or fractional).
- 3) Additivity: Additivity means that the value of the objective function for the given value of decision variables and the total sum of resources used, must be equal to the sum of the contributions (Profit or Cost) earned from each decision variable and sum of the resources used by each decision variable respectively. The objective function is the direct sum of the individual contributions of the different variables.
- 4) Linearity: Linearity means that all relationships in the LP model (i.e. in both objective function and constraints) must be linear.

3. Synthetic Fertilizers Industry in Jordan

Synthetic fertilizers and pesticides industry is a form of secondary chemical production. It is a manufacturing sub-sector. According to the records of the Jordanian Department of Statistics (DoS) there are more than 60 plants in this sub-sector (DoS, 2014). Table 1 shows total production of synthetic fertilizers in Jordan in the last six years. Around 33% of production of these plants is consumed by the agricultural sector. Table 2 shows consumption of synthetic fertilizers in agriculture. Synthetic fertilizers and pesticides manufacturing subsector contribution to the gross domestic production (GDP) is 0.45%. This subsector rank in contribution to GDP is 41 out of nearly 80 other industrial subsectors, and its rank in the value-added from this subsector is 16 among other manufacturing subsectors with a percentage of 2.6% of total value-added from these manufacturing industries. Synthetic fertilizers and pesticides industry share in national exports is estimated to be 2.7% (DoS, 2014).

Table 1. Total production of synthetic fertilizers in Jordan (2009 - 2014)

Year	Total Production (1000 Tons)
2009	721.1
2010	759.9
2011	722.7
2012	640.3
2013	678.1
2014*	238.5

Source: Central Bank of Jordan (CBJ), March 2014.

*Only for January, February, and March.

Table 2. Consumption of synthetic fertilizers in agriculture;

Type	Consumption (%)
Vegetables	19.72
Fruit Trees	6.23
Agrochemicals	4.53
Field crops	1.64
Other	0.72
Total	33.24

Source: DoS, 2014.

4. Materials and Methods

4.1 Data

Data about fertilizers specifications and constraints imposed on the fertilizers components were collected from 30 synthetic fertilizers plants. Costs of ingredients used in fertilizers formulation were obtained from the prevailing market prices. Secondary data sources were mainly the Department of Statistics, Ministry of Agriculture Directorates, and other related sources (published studies, bulletins, books ...).

4.2 Data Analysis

Linear Programming models were constructed in this study to reflect various ingredients combinations used in fertilizers formulation, prevailed market prices, ingredients composition, and range of inclusion to obtain a least-cost fertilizer mix. To achieve the objectives of the study, an objective function is minimized subject to certain constraints. Here, the objective function is a cost function representing the total cost of ingredients used to formulate the mix.

4.3 Model Construction

Linear programming models were constructed for three N-P-K combinations. These were; 20-20-20, 12-12-36, and 15-30-15. Trace elements were added to the three combinations as required. N-P-K is used to label fertilizer based on the relative content of the elements nitrogen, phosphorus, and potassium. The N value is the percentage of elemental nitrogen by weight in the fertilizer. The values for P and K represent the amount of oxide in the form of P₂O₅ and K₂O that would be present in the fertilizer if all the elemental phosphorus and potassium were oxidized into these forms.

The objective of the models was to minimize cost of producing a particular fertilizer mix after satisfying a set of constraints. To compare costs and to determine the least cost mix, three types of synthetic fertilizers mixes were formulated. The variables in the models were the ingredients. Table 3 shows these ingredients and their prices, and Table 4 shows constraints imposed on the selection of ingredients according to NPK label. The labels on fertilizer packages give the amount of the three Primary Nutrients (nitrogen, phosphorous, and potassium), expressed as a percentage of the total fertilizer weight. The format used to display this information is the "N-P-K" label, on which the first number is the percentage of nitrogen, the second number is the percentage of

phosphorus (in the form of P_2O_5 , or phosphate), and the third number is the percentage of potassium (in the form of K_2O , or potash). For example NPK label of 21-0-0 means 21% nitrogen, 0% phosphorus, and 0% potassium, and 10-20-20 NPK label means 10% nitrogen, 20% phosphorus, 20% potassium. Constraints imposed on fertilizers mix according to NPK label are shown in Table 5. The selection of these three blends was made upon farmers' opinions.

Table 3. Ingredients used in LP models and their costs

Ingredient	Name	Price (JDs*/ton)
X1	Mono Ammonium Phosphate (MAP)	910
X2	Mono Potassium Phosphate (MKP)	1200
X3	Potassium Nitrate (PN)	825
X4	Ammonium Sulfate (AS)	280
X5	Ammonium Nitrate (AN)	550
X6	Urea Phosphate (UP)	350
X7	Potassium Sulfate (PS)	500
X8	Trace Elements (TE)	3000
X9	processed urea (PU)	260

Source: Study Survey. *1 JD = 1.4 USD.

Table 4. Constraints imposed on the selection of f according to NPK label

Ingredient	NPK Label
X1	12-61-0
X2	0-52-34
X3	13-0-46
X4	21-0-0
X5	33-0-0
X6	17-44-0
X7	0-0-50
X8*	*****
X9	46-0-0

Source: Study Survey.

*Trace Elements = 4%.

Table 5. Constraints imposed on fertilizers mix according to NPK label

Ingredient	NPK Label
Mix 1	20 – 20 - 20
Mix 2	15 – 30 - 15
Mix 3	12 – 12 - 36

Source: Study Survey.

The specified linear programming model for the attainment of the objective function is:

$$\text{Minimize } Z = \sum C_{ij} X_i$$

Where, Z = Total cost of fertilizer mix; C_{ij} = Ingredient cost; X_i = Ingredient quantity.

The general mathematical form of the models was as follows;

$$\text{Minimize } Z = C_1 X_1 + C_2 X_2 + \dots + C_n X_n$$

Subject to constraints,

$$a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n \leq b_1$$

$$a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n \leq b_2$$

$$a_{31}X_1 + a_{32}X_2 + \dots + a_{3n}X_n \leq b_3$$

$$a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n \leq b_m$$

and $X_1, X_2, \dots, X_n \geq 0$

Where, a_i = Technical coefficients of mix components; b_i = constrains of the mix.

2.4 LP Models

(1) LP model for mix 1 (20 – 20 – 20) is,

$$\text{Min } (Z) = 910X_1 + 1200X_2 + 825X_3 + 280X_4 + 550X_5 + 350X_6 + 500X_7 + 3000X_8 + 260X_9$$

Subject to,

$$12X_1 + 13X_3 + 21X_4 + 33X_5 + 17X_6 + 46X_9 \leq 200;$$

$$61X_1 + 52X_2 + 44X_6 \leq 200;$$

$$34X_2 + 46X_3 + 50X_7 \leq 200;$$

$$X_8 = 4;$$

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 = 1000;$$

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9 \geq 0.$$

(2) LP model for mix 2 (15 – 30 – 15) is,

$$\text{Min } (Z) = 910X_1 + 1200X_2 + 825X_3 + 280X_4 + 550X_5 + 350X_6 + 500X_7 + 3000X_8 + 260X_9.$$

Subject to,

$$12X_1 + 13X_3 + 21X_4 + 33X_5 + 17X_6 + 46X_9 \leq 150;$$

$$61X_1 + 52X_2 + 44X_6 \leq 300;$$

$$34X_2 + 46X_3 + 50X_7 \leq 150;$$

$$X_8 = 4;$$

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 = 1000;$$

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9 \geq 0.$$

(3) LP model for mix 3 (12 – 12 – 36) is,

$$\text{Min } (Z) = 910X_1 + 1200X_2 + 825X_3 + 280X_4 + 550X_5 + 350X_6 + 500X_7 + 3000X_8 + 260X_9.$$

Subject to,

$$12X_1 + 13X_3 + 21X_4 + 33X_5 + 17X_6 + 46X_9 \leq 120;$$

$$61X_1 + 52X_2 + 44X_6 \leq 120;$$

$$34X_2 + 46X_3 + 50X_7 \leq 360;$$

$$X_8 = 4;$$

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 = 1000;$$

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9 \geq 0.$$

5. Results and Discussion

The optimum linear programming solutions for the three combinations of the investigated synthetic fertilizers according to NPK label and their compositions are shown in Tables 6, 7, and 8. The three combinations were;

20-20-20, 15-30-15, and 12-12-36. These three combinations (see Table 5 please) were selected to be used as a constraints in the model according to their importance to plants. The first combination, which may be called the balanced combination (20-20-20), was selected due to its importance in increasing the shoot and the root, as well as, the fruit development in a balanced manner. The second combination (15-30-15) was selected due to its main role in root development during the early stages of plant life. The high P content of this combination plays a critical role in root development which helps in anchoring the plant and enabling it to absorb nutrients and water from the soil efficiently. The third combination (12-12-36) with high K content was selected due to its main role in increasing cellular division in the fruit. The least cost combinations of the three mixes of the synthetic fertilizers according to the proposed constraints shown in Tables 6-8.

Table 6. Optimum LP solution for mix 1

Ingredient	KGs	%N	%P	%K	Price (JDs*/ton)
X1	*****	*****	*****	*****	*****
X2	278.8	*****	14.5	9.48	317.89
X3	54.77	0.76	*****	2.52	45.190
X4	7.760	0.16	*****	*****	2.0200
X5	*****	*****	*****	*****	*****
X6	125.0	2.13	5.50	*****	100.00
X7	160.0	*****	*****	8.000	77.600
X8	4.000	*****	*****	*****	*****
X9	369.57	17.00	*****	*****	129.35
Total	1000	20	20	20	672

Table 7. Optimum LP solution for mix 2

Ingredient	KGs	%N	%P	%K	Price (JDs*/ton)
X1	491.8	5.900	30.0	*****	450.00
X2	*****	*****	*****	*****	*****
X3	*****	*****	*****	*****	*****
X4	4.680	0.10	*****	*****	1.2200
X5	14.71	0.50	*****	*****	8.0900
X6	*****	*****	*****	*****	*****
X7	300.0	*****	*****	15.00	145.50
X8	4.000	*****	*****	*****	*****
X9	184.78	8.500	*****	*****	64.670
Total	1000	15	30	15	669

Table 8. Optimum LP solution for mix 3

Ingredient	KGs	%N	%P	%K	Price (JDs*/ton)
X1	49.18	0.59	3.00	*****	45.000
X2	163.46	*****	8.50	5.56	186.34
X3	455.27	5.92	*****	20.94	375.60
X4	14.20	0.30	*****	*****	3.6900
X5	14.71	0.50	*****	*****	8.0900
X6	11.36	0.19	0.50	*****	9.0900
X7	160.0	*****	*****	9.50	92.150
X8	4.000	*****	*****	*****	*****
X9	97.83	4.50	*****	*****	34.240
Total	1000	12	12	36	754

Table 6 shows the least cost of the balanced combination 20-20-20. The least total cost of producing one ton of this combination, among other combinations, was 672 JDs (1 JD = 1.4 USD). The ingredients of this combination included 278.8, 54.77, 7.79, 125, 160, 4, and 369.57 kgs of X2, X3, X4, X6, X7, X8, and X9 respectively. Table 7 shows the least cost of the combination 15-30-15. The least total cost of producing one ton of this combination, among other combinations, was 669 JDs. The ingredients of this combination included 491.8, 4.68, 14.71, 300, 4, and 184.78 kgs of X1, X4, X5, X7, X8, and X9 respectively. Table 8 shows the least cost of the combination 12-12-36. The least total cost of producing one ton of this combination, among other combinations, was 754 JDs. The ingredients of this combination included 49.18, 163.46, 455.27, 14.2, 14.71, 11.36, 160, 4, and 97.83 kgs of X1, X2, X3, X4, X5, X6, X7, X8, and X9 respectively.

6. Conclusion

The least cost synthetic fertilizer combinations of the three studied fertilizer mixes among many calculated combinations produced by the linear programming technique was 672 JDs for the first mix with NPK label of 20-20-20, 669 JDs for the second mix with NPK label of 15-30-15, and 754 JDs for the third mix with NPK label of 12-12-36. These all three costs are lower by nearly 5-10% than those imposed by the market. These results confirm the importance of using techniques such as LP to formulate least cost products in industries such as synthetic fertilizers industry.

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