THE COSMIC MICROWAVE BACKGROUND

The cosmic microwave background (CMB) is the radiation left over from the big bang. Recent analysis of the fluctuations in this radiation has given us valuable insights into our Universe and its parameters.

This textbook examines the theory of CMB and its recent progress. It starts with a brief introduction to modern cosmology and its main successes, followed by a thorough derivation of cosmological perturbation theory. It then explores the generation of initial fluctuations by inflation. In the following chapters the Boltzmann equation, which governs the evolution of CMB anisotropies, and polarization are derived using the total angular momentum method. Cosmological parameter estimation is discussed in detail. The lensing of CMB fluctuations and spectral distortions are also treated.

The book is the first to contain a full derivation of the theory of CMB anisotropies and polarization. Ideal for graduate students and researchers in this field, the textbook includes end-of-chapter exercises, and solutions to selected exercises are provided.

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To Martin, Florian, Melchior and Anna

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Preface

Cosmology, the quest concerning the Universe as a whole, has been a primary interest of human study since the beginnings of mankind. For a long time our ideas about the Universe were dominated by religious beliefs – tales of creation. Only since the advent of general relativity in 1915 have we had a scientific theory at hand that might be capable of describing the Universe. Soon after Einstein's first attempt of a static universe, Hubble and collaborators (Hubble, 1929) discovered that the observable Universe is expanding. This together with the discovery of the cosmic microwave background (CMB) by Penzias and Wilson (Nobel prize 1978) has established the theory of an expanding and cooling universe which started in a 'big bang'.

For a long time observations that have led to the determination of cosmological parameters, such as the rate of expansion, the so-called Hubble parameter, the mean matter density of the Universe, or its curvature, have been very sparse and we could only determine the order of magnitude of these parameters.

During the last decade this situation has changed significantly and cosmology has entered an era of precision measurements. This major breakthrough is to a large extent due to precise measurement and analysis of the CMB. In this book I develop the theory which is used to analyse and understand measurements of the CMB, especially of its anisotropies and polarization, but also its frequency spectrum. The Nobel prize was awarded to George Smoot and John Mather, in 2006, for the discovery of these anisotropies and for precise measurements of the CMB spectrum.

The book is directed mainly towards graduate students and researchers who want to obtain an overview of the main developments in CMB physics, and who want to understand the state-of-the-art techniques which are used to analyse CMB data. I believe that the theory of CMB physics is now sufficiently mature for a book on this topic to be useful. I shall not enter into any details concerning CMB experiments. This is by no means because I consider them less interesting, but rather that they are still in full development and will hopefully make significant progress in the near future. Of course, my background is also that of a theoretical physicist and my main interest lies in the theoretical aspects of CMB physics. I hope, however, that this book will also be useful to CMB experimentalists who want to know what happens inside their cosmic parameter estimation routines.

It is assumed that the reader is familiar with undergraduate physics including the basics of general relativity, and has an elementary knowledge of quantum field theory and particle physics. The beauty of cosmology lies in the fact that it employs more or less all fields of physics starting with general relativity over thermodynamics and statistical physics to electrodynamics, quantum mechanics and particle physics. In this book I do not want to present an introduction to these topics as well since, first of all, there exist wonderful textbooks on all of them and second you have learned them in your undergraduate physics courses.

Before we start, let me sketch the content of the different chapters and give you a guide on how to read this book.

The first chapter is an overview of the homogeneous and isotropic universe. We present and discuss the Friedmann equations, recombination, nucleosynthesis and inflation. Readers familiar with cosmology may skip this chapter or just skim it.

In Chapter 2 we develop cosmological perturbation theory. This is the basics of CMB physics. The main reason why the CMB allows such an accurate determination of cosmological parameters lies in the fact that its anisotropies are small and can be determined within first-order perturbation theory. In Fourier space the linear perturbation equations become a series of ordinary linear differential equations, which can be solved numerically to high precision without any difficulty. We derive the perturbations of Einstein's equations and the energy–momentum conservation equations and solve them for simple but relevant cases. We also discuss the perturbation equation for light-like geodesics. This is sufficient to calculate the CMB anisotropies in the so-called instant recombination approximation. The main physical effects which are missed in such a treatment are Silk damping on small scales and polarization. We then introduce the CMB power spectrum and draw our first conclusions for its dependence on cosmological and primordial parameters. For example, we derive an approximate formula for the position of the acoustic peaks. An experimentalist mainly interested in parameter estimation may jump, after Chapter 2, directly to Chapter 6 and skip the more theoretical parts between.

The third chapter is devoted to the initial condition. There we explain how the unavoidable quantum fluctuations are amplified during an inflationary phase and lead to a nearly scale-invariant spectrum of scalar and tensor perturbations. We also discuss the initial conditions for mixed adiabatic and iso-curvature perturbations.

In Chapter 4 we derive the perturbed Boltzmann equation for CMB photons. After a brief introduction to relativistic kinetic theory, we first derive the Liouville

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equation, i.e. the Boltzmann equation without a collision term. We also discuss the connection between the distribution function and the energy–momentum tensor. We then derive the collision term, i.e. the right-hand side of the Boltzmann equation, due to Thomson scattering of photons and electrons. In this first attempt we neglect the polarization dependence of Thomson scattering. The chapter ends with a list of the full system of perturbation equations for a ΛCDM universe.

In Chapter 5 we discuss polarization. Here we derive the total angular momentum method that is perfectly adapted to the problem of CMB anisotropies and polarization, taking into account its symmetry, which allows a decomposition into modes with fixed total angular momentum. The representation theory of the rotation group and the spin weighted spherical harmonics which are extensively used in this chapter are deferred to an appendix. We interpret some results using the flat sky approximation, which is valid on small angular scales.

Chapter 6 is devoted to parameter estimation. We first discuss the physical dependence of CMB anisotropies on cosmological parameters. After a section on CMB data we then treat in some detail statistical methods for CMB data analysis. We discuss especially the Fisher matrix and explain Markov chain Monte Carlo methods. We also address degeneracies, combinations of cosmological parameters on which CMB anisotropies do not, or only very weakly, depend. Because of these degeneracies, cosmological parameter estimation also makes use of other, non-CMB related, observations. We summarize them in a separate section. We finish the chapter with a discussion of 'sources', i.e. inhomogeneously distributed contributions to the energy–momentum tensor, such as topological defects, which may also contribute to the CMB anisotropies and thereby affect the estimated cosmological parameters.

In Chapter 7 we treat lensing of CMB anisotropies and polarization. This second-order effect is especially important on small scales but also has to be taken into account for $\ell \gtrsim 500$ if we want to achieve an accuracy of better than 0.5%. We first derive the deflection angle and the lensing power spectrum. Then we discuss lensing of CMB fluctuations and polarization in the flat sky approximation, which is sufficiently accurate for angular harmonics with $\ell \gtrsim 50$. We conclude the chapter with an overview on other second-order effects.

In the final chapter spectral distortions of the CMB are discussed. We first introduce the three relevant collision processes in a universe with photons and non-relativistic electrons: elastic Compton scattering, Bremsstrahlung and double Compton scattering. We derive the corresponding collision terms and Boltzmann equations. For elastic Compton scattering this leads us to the Kompaneets equation for which we present a detailed derivation. We introduce the timescales corresponding to these three collision processes and determine at which redshift a given process freezes – becomes slower than cosmic expansion. Finally, we discuss the possible generation of a chemical potential in the CMB spectrum and the Sunyaev–Zel'dovich effect.

All chapters are complemented with some exercises at the end.

In the appendices we collect useful constants and formulae, information on special functions and some more technical derivations. The solutions to a selection of exercises are also given in an appendix.

This book has grown out of a graduate course on CMB anisotropies that I have given on several occasions. Thanks are due to the students of these courses, who have motivated me to write it up in the form of a textbook. I am also indebted to many collaborators and colleagues with whom I have discussed various aspects of the book and who have helped me to clarify many issues. Especially I want to mention Chiara Caprini, Martin Kunz, Toni Riotto, Uros Seljak and Norbert Straumann. I am also immensely grateful to students and colleagues who have read parts of the draft and helped me correct numerous typographical errors and other mistakes: Camille Bonvin, Jean-Pierre Eckmann, Alice Gasparini, Sandro Scodeller and others. Of course all the remaining mistakes are entirely my responsibility. Marcus Ruser and Martin Kunz have also helped me with some of the figures. I also wish to thank Susan Staggs who provided me with a most useful dataset of the CMB spectrum.

Ruth Durrer

The cosmic microwave background (CMB, CMBR), in Big Bang cosmology, is electromagnetic radiation as a remnant from an early stage of the universe, also known as "relic radiation". The CMB is faint cosmic background radiation filling all space. It is an important source of data on the early universe because it is the oldest electromagnetic radiation in the universe, dating to the epoch of recombination. With a traditional optical telescope, the space between stars and galaxies (the background) is Cosmic microwave background (CMB), also called cosmic background radiation, electromagnetic radiation filling the universe that is a residual effect of the big bang 13.8 billion years ago. Because the expanding universe has cooled since this primordial explosion, the background radiation is in the microwave region of the electromagnetic spectrum. Discovery of the cosmic background.Â Precise measurements made by the Cosmic Background Explorer (COBE) satellite launched in 1989 determined the spectrum to be exactly characteristic of a blackbody at 2.735 K. The velocity of the satellite about Earth, Earth about the Sun, the Sun about the Galaxy, and the Galaxy through the universe actually makes the temperature seem slightly hotter (by about one part in 1,000).

The â€˜Cosmic Microwave Background radiationâ€™ (CMB) is the record of these photons at the moment of their escape.Â The photons of the CMB were emitted at the epoch of recombination when the Universe had a temperature of about 3,000 Kelvin. However, they have been cosmological redshifted to longer wavelengths during their ~13 billion year journey through the expanding Universe, and are now detected in the microwave region of the electromagnetic spectrum at an average temperature of 2.725 Kelvin. This agrees well with what Big Bang theory predicts.