THE CHALLENGE OF TEACHING THE MESOAMERICAN NUMBERS: ALIGNING WITH INDIGENOUS-MATHEMATICAL THOUGHT.

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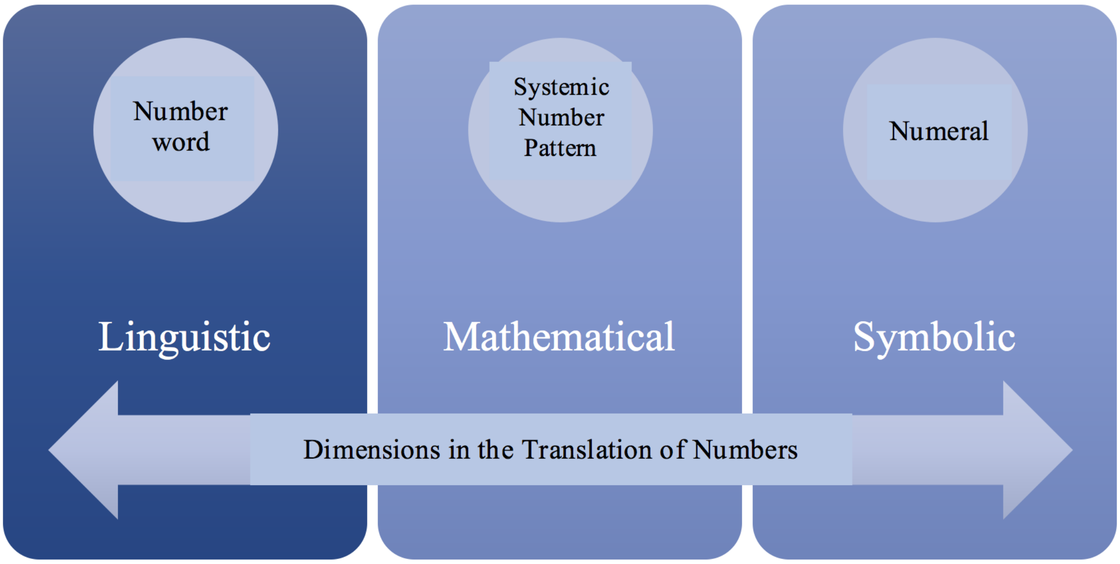
Mathematics and language are intricately related. Language helps us name the ideas and object we think about and deal with in any context; Mathematics is not an exception. Mathematical concepts and ideas are named and manipulated through language. In the Ask Dr. Math, Math Forum (NCTM, 2017), numbers are defined as abstract concepts, mathematical objects, constructions within a numerical system that help us count in an organized sequence. Numbers are represented through symbols or numerals. With the goal of clarifying the distinction between numbers and numerals, Dr. Peterson presents a description in the forum: “the number 5 can be thought of as the concept of "fiveness" which all sets of five objects have in common; it can be expressed using numerals such as 5, V, |||||, five, 101 (base-2), and so on” (NCTM, 2017, second paragraph). Consequently, numbers as abstract concepts of quantities like “fiveness” need a visual representation linked to that concept, for this we use the numeral, “5”. Furthermore, when extending these ideas to language, especially thinking of naming numbers across languages we could add another dimension, the name of the “fiveness” in different languages. Therefore, in Spanish, we could have these three levels or dimensions of a number: *cinco*, 5, and the abstract idea of “fiveness”. In German, *fünf*, 5, and the abstract idea of “fiveness”. However, when thinking of different number systems, as Dr. Peterson presented, I argue that there is more to think that the idea of “fiveness”. The numeral “101” in a binary system, indeed represents mathematically the abstract idea of “fiveness”, but this idea of “fiveness” is deconstructed through a numerical system of base-2. Where the words or name for this numeral is “one, zero, one”; and in which the ones in third and the first numerical positions from the left are stand for “fourness” plus “oneness” and it also implies the absence of “twoness”. I argue that across numerical systems the idea of “fiveness” is re-constructed, transformed into different assumptions to match the idea of “fiveness”. This kind of translation, or rather interpretation also happens across languages, where different assumptions about life are made. For example, in English one “is” certain numbers years old: *Marta is five years old*. In Spanish, one “has” certain numbers of years: *Marta tiene cinco años*. In other languages and cultures the idea of measuring the age of person might not even exist. So, when translating these ideas across languages ideas need to be re-accommodated, interpreted to match or be understood in the new linguistic and cultural system. Across numbers systems, the idea or concept of number varies and its conversions will depend on the rules and assumptions of the numerical systems in contact. Thus, to elaborate on the process of linking numbers across systems and languages, I argue that to process or translate numbers across languages and systems three dimensions need to be addressed.

Figure 1—Dimensions to translate numbers across languages and numerical systems.



Using "35" as an example, during translation of numbers across numerical systems (from decimal in English to vigesimal in Ixil), both the original symbolic representation (35) and the naming (thirty-five) of a number need to be understood within a system of mathematical patterns and relationships that determine the value of that number (thirty-fiveness which in a decimal system equals three tens and five ones). Then, the initial mathematical pattern of that number (thirty-fiveness or three tens and five ones) needs to reinterpreted into the new system (base-20 system, which is fifteen ones and a group of twenty, or fifteenness plus one twentyness), this new organization yields the symbolic representation (≐) and naming (Olavaj ika'k'al) of a pattern or number equivalent to the original number concept (thirty-fiveness).

In this paper, using these three dimensions, I present how traditional teaching practices in mathematics education limit the process of developing a solid understanding of other numerical systems, specifically of the Mesomerican (base-20) numerical system. Such practices undermine not only the academic goals of nurturing mathematical reasoning and understanding, but also the intercultural education goals of promoting and valuing indigenous cultures, languages, and epistemologies (Bryan, 2006; Grisby, 2004; López, 2009). Alternatively, I describe how the inclusion of these three dimensions during teaching may support more robust understanding of mathematical concepts and indigenous languages through the teaching and learning of Mesoamerican numbers. For this, I describe ways of teaching the Mesoamerican numbers to both children (Ixil speakers) in Guatemala, and bilingual adults (English and Spanish speakers) in the U.S. Learners understood Mesoamerican numbers better when intrinsic mathematical paradigms in the language (Ixil) and numerical representations were used as generative knowledge.

Mesoamerica

Mesoamerica, "middle America", is an area that extends from approximately central Mexico through Belize, Guatemala, El Salvador, Honduras, Nicaragua, and northern Costa Rica. The Mesoamerican civilization encompasses groups of peoples with close cultural and historical ties. There is over 11 million of indigenous people, descendants of pre-Columbian cultures, who represent approximately 17.2% of total population of this area. The pre-Columbian Mesoamerican civilizations include the Olmec, Teotihuacan, Zapotec, Maya, Huastec, Purepecha, Toltec, Mixtec, and Mexica/Aztec (OECD, 2006). Guatemala the indigenous populations constitute 40% of the total population (López, 2009).

Intercultural Bilingual Education

Intercultural bilingual education (IBE) in Latin America emerged as an alternative to monolingual instruction of westernized values and beliefs, so that needs, epistemologies and interests of indigenous populations work as axes of the school curriculum (Bryan, 2006; Grisby, 2004; López, 2009). In Guatemala, specifically, wherein the work reported here took place, there are 7,832 public elementary schools, from those 76% (5,963 schools) are monolingual in Spanish and only 24% (1,869 schools) officially provide IBE (Bryan, 2006). IBE in Guatemala is linked to the framework of the peace agreements signed in 1996. This framework highlights the intercultural rather than the multicultural goal of education given the diversity of the population and its need of reconciliation and dialogue. As mentioned, IBE centers on the indigenous population needs and interests since in Guatemala present and future generations are often prohibited from self-identifying with their ethnic, cultural and linguistic background without stigma. Consequently, IBE deliberately challenges ethnic, social or sexual stereotypes and discrimination, and instead promotes multiple languages, cultures, and epistemologies (Grisby, 2004). IBE supports explicitly the teaching of Mayan[1] mathematics since the framework recognizes the relationship between language and thought as well as between language and culture. The learning of the vigesimal, base-20, system is considered as a knowledge that will empower learners by deepening their reasoning and solution of problems. The fluid movement between decimal and vigesimal systems is also linked to the country's cultural plurality. Students are to learn five sub-areas in Mayan Mathematics including: "philosophy of numbers, numbering, arithmetic, measurement and mathematical investigation. The sub-areas are organized into conceptual contents that include those referring to mathematics, knowledge and their interrelationships" (Grisby, 2004, p. 67).

Mayan Languages in Guatemala: Ixil

Guatemala has 23 different languages besides Spanish (the official language). 21 of these languages are Mayan-related, two (Xinca and Garifuna) have diverse origins connected to El Salvador and the Caribean coast. The worked referenced in this paper was took place at a school in a town in The Ixil region in Guatemala, which includes three main areas: Nebaj, Chajul and Cotzal. The name Ixil relates to the name of the language and ethnic group of people living in this area. Ixil is a language of Mayan origin. Historically, the Ixil people were paid very low salaries and forced to work on the coastal plantations. Many people have migrated to the coast, Guatemala City and the US in search of work. During the Guatemalan violent conflict this area was severely affected and became the main operation center for the Guerrilla Army. After peace agreements were signed, many villagers returned to their land (Rough Guides, n.d.).

The village that hosts the school is one and a half hours away from the nearest town. The school curriculum aligns with the Guatemalan content standards. The mathematics curriculum includes a textbook that each student owned. The classroom was 5th grade with fifteen children and teacher are from a Mayan background and bilingual (Spanish and Ixil). Six of them are female and the rest male. Most children studied at this school since first grade and all lived in the village. The researcher, was an *outsider* researcher Ladino, bilingual (Spanish and English), teacher educator, of Guatemalan origin, who works in the field of bilingual mathematics education in the United States. The teacher and researcher co-taught for a week the mathematics classes in Spanish. The teacher taught independently and bilingually the mathematics lessons

[1] While the numbers described here, vigesimal mathematics system, are often deemed as Mayan. However, these numbers can be recognized as Mesoamerican as other civilizations in the area also used them.

in Ixil and helped the outsider research keep up with the pace of the group of students. In addition, I also taught bilingually (Spanish and English) these numbers to adult students in graduate and undergraduate programs in a course named Language, Culture, and Mathematics. While Ixil language or words of the numbers were presented linked to numerical representations, none of these students spoke an indigenous language.

Researcher's Positionality

As an outsider researcher and a mathematics educator, I understand the main goal of my work focusing on mathematics teaching and learning practices that facilitate deep student learning. I am mestizo and I do not speak an indigenous language. On the one hand, I want to express the love towards Guatemala, my country even though I do not reside in it. All my relatives live there. I taught in different grades and subject between K-11 grades during 15 years. I live in the U.S. and have a deep commitment to bilingual mathematics education and ethnomathematics approaches (D'Ambrossio, 1985) since they place emphasis on the learners' interests, needs, language and culture first, while promoting access to mainstream languages and mathematics. Therefore, my goal in this study is twofold. First, I recognize the value of indigenous epistemologies, languages, and mathematical knowledge in the intercultural bilingual education of indigenous learners, and in how schools, especially IBE schools, need to implement pedagogical approaches that nurture the development of understanding and use of Mesoamerican mathematics. Second, I focus on the development of understanding across linguistic number systems by capitalizing on the relations of language, thought, and mathematics. For this, I draw from my experiences with 5th grade bilingual students (Ixil and Spanish) and education-major adult college students in the U.S.; some of whom were bilingual (Spanish and English) and some monolingual (English) speakers.

Theoretical Frame

Given the intercultural nature of the problem in this study and its focus on the link between language, thinking and mathematics, I draw on Vigotski's (2012) idea of the development of thought and human consciousness through the union of thought and language during the process of understanding the meaning of the word. Vigotski argues that thought and language are not linked by an organic connection; this link emerges, it transforms and grows through the development of thinking and the development of the meaning of the word. The meaning of the word encompasses the indivisible union of thought and word. Vigotski argues:

> "*El significado de la palabra es un fenómeno del pensamiento solo en la medida en que el pensamiento esté relacionado con la palabra, y encarnado en ella, y a la inversa: es un fenómeno del habla solo en la medida en que el habla esté relacionada con el pensamiento e iluminada por él* " / The meaning of the word is a phenomenon of the thought only in the way that the thought is related with the word, and is incarnated in it, and vice-versa: it is a phenomenon of the speech only in the way that the speech is related to the thought and be enlightened by it (p. 426, my translation).

From this perspective in mathematics, the meaning of the word or the meaning of number "fiveness" exists when the thought about "fiveness" is linked the word "five" and when speaking the word "five" is linked to the thought of "fiveness". As I have tried to describe above, the meaning of "fiveness" or a number needs to be linked to a system of mathematical patterns and concepts that give order or a unique position and a mathematical value to a specific number. The meaning/understanding of the word or number is dependent on the understanding of the numerical system. Numbers are meaningful through the system that they belong to. Isolated verbal and visual representations of numbers are just sounds and forms that could be interpreted in multiple ways. The mathematical nature of numbers is rooted in the mathematical concepts, norms, and assumptions in each system of numbers that organizes and regulates numbers. The word then stands for an expression of the though. Numbers are expressed both by words (in a language system—like Spanish) and symbolic representations, or numerals. For the numerals and number names to be meaningful need to be connected to the understanding of the idea or concept of what a number means. The established relationship among "5", "cinco" and "fiveness" would support an understanding of the meaning of the number five.

The process of understanding, or developing meaning of the word/number, is developed through the process of how thought and the word align with and inform each other. Vigotski asserts:

> *La relación entre el pensamiento y la palabra no es una cosa, sino un proceso, el movimiento del pensamiento hacia la palabra y, a la inversa, de la palabra hacia el pensamiento,* […] *Por cierto, no se trata del desarrollo de la edad, sino del funcional, pero el movimiento del propio proceso del pensar, del pensamiento a la palabra, es desarrollo. El pensamiento no se refleja en la palabra, sino que se realiza en esta*. / The relationship between thought and the word is nothing else, but a process, the movement of the thought towards the word and, vice-versa, from the word towards the thought, […] In fact, it is not about the development of age, but of a functional one, but of the movement of the thinking itself, of the thought to the word, is the development. The thought is not reflected in the word, but it is realized within it. (p. 438, my translation)

The relationship between thought and word or thought and number develops as the process the movement between thought and number concepts come together and one is can think with numbers. Therefore, in this paper I am focusing on exploring the relationship between thought and number as the movement of thought towards understanding the conceptual mathematical relationship of numbers across numerical systems and languages. The example of Ixil and English languages, and vigesimal and decimal systems support this analysis. I understand language, as a code, as it is linked to and embeds mathematical paradigms of a specific system of numerical thought.

Number Systems: Mesoamerican Numbers

Humans, inherently mathematical, have developed multiple numerical systems around the globe, such as base-2, base-10, base-16, base-20, base-60, etc. (Mastin, 2010b) that help describe, count, measure, shape, or program objects important for social, cultural, and academic practices. The decimal number system has become the default number system and often influences how people understand numbers. In number 68 (sixty-eight), the six is worth six groups of ten, and the eight, eight ones. Under this paradigm, other numerical systems (e.g., binary, sexagesimal) might seem confusing. For example, in the decimal system the symbols: 1000_{10} and 1010_{10} respectively represent values of one thousand and one thousand and ten units. Contrastingly, in the binary numbers the same symbols represent the value of eight units for 1000_2 and ten units for 1010_2. While in English the naming of the binary numbers is not directly linked to their position, in other number systems the link to language is evident. I think it is because the binary numbers were designed to match the processing of computers, so that the inputs of electricity or absence of it could be interpreted to encode and decode information. To ease the contrast of the decimal and binary numbers, Tables 1 and 2 below describe how the value of the number 68 is represented in each system. Decimal numbers have 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Binary numbers have 2 digits: 0 and 1. Each of the digits represents the units that are included, their position in the columns determine how much of each of those units is worth. For example, 68 as written here related to a decimal numerical system. As Table 1 shows, the 6 is under the position where each unit is worth a ten, so the 6 implies six tens or groups of ten. The 8 because of its position, represents eight ones. Then, the number 68 in decimal system is understood as six tens and eight ones.

Table 1: Positional value in the decimal system.

Base-10 exponentials	10^5	10^4	10^3	10^2	10^1	10^0
Equivalent Decimal	**100,000**	**10,000**	**1,000**	**100**	**10**	**1**
English words	Hundred thousands	Ten thousands	thousands	hundreds	tens	ones
Sixty-eight					6	8

Contrastingly, in the binary system the number 68 is understood or decomposed in different pattern of groups. Table 2 seems less familiar to us than Table 1. Binary numbers with only two digits, the values of units grow exponentially in powers of two. So, in the first column (2^0) the digits would represent one or zero. In the second column (2^1), the digits will represent either two (2 x 1=2) or zero. In the third column (2^2), the digits will represent either four (2x2=4) or zero. The fourth column (2^3), the values will be eight (2x2x2=8) or zero. And so forth with the following positions. Therefore, to represent the number 68 in binary numbers, this number must be understood within a system of power of two. The closest value to 68 in Table 2 is under the column in the 7th position or "2^6", so if we place the digit 1 in this position, the number indicates that the value of sixty-four ones is accounted for once. Since more ones are needed to complete the value of 68, we can see that the four ones that are still missing could be completed by the value in the third column or "2^2". By adding the digit 1 under this column we add the value of four ones once to the value of sixty-four ones that we had before, so together these two positions provide the value of sixty-eight ones, then the rest of positions or columns will have the digit "0" which means that the sets in those positions are not activated. Thus, the binary number: "1000100" represents the number "68".

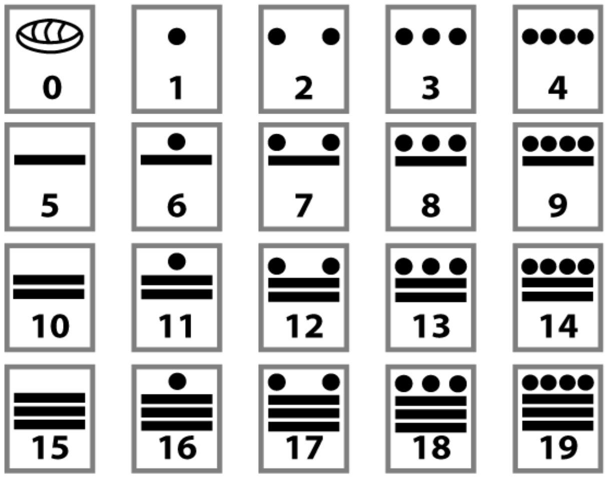
Table 2: Positional value in the binary system.

Base-2 exponentials	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Equivalent Decimal	**64**	**32**	**16**	**8**	**4**	**2**	**1**
English words	Sets of sixty-eight	Sets of thirty-two	Sets of sixteen	Sets of eight	sets of four	Set of two	ones
Sixty-eight	1	0	0	0	1	0	0

Notice how helpful the decimal numbers and values of sets described in decimal numbers become for us to more successfully manipulate or encode a value in a binary numerical system. Without that support or translation, the digits 1 and 0 seems meaningless. And if we read them with the mathematical convention of a decimal system, we would read the number: one million one hundred (i.e., $1{,}000{,}100_{10}$), and with difficulty we would see the "sixty-eightness" of the number 68.

But now, let's move onto the Mesoamerican numbers, commonly referred to as Mayan numbers. Frequently, the Mesoamerican numbers, since 36 BCE (Mastin, 2010a), are easily recognized by its representations (stick, stones, shells). Numbers, 0-20, are commonly identified through original representations: stick (line segment) = 5 units, stone (dot) = 1 unit, and shell= zero; with a maximum of three lines and four dots, one can represent the digits of 0-19 (Díaz Díaz, 2006).

Figure 2: Mesoamerican digits



As seen in Figure 2, the same representations (stick, stones, shells) are combined to form the 20 digits. For numbers greater than 19, these digits can take up any position to represent any number one can imagine. Continuing with number 68, in a vigesimal system such as the Mesoamerican numbers, this number needs to be understood in base-20, so in the value of sixty-eight there are three twenties and eight extra ones. For this reason, Table 3 depicts the digit three under the second column (20^1) for a value of sixty units (20x1=20x3=60), then the digit eight under the first column (20^0) represents eight ones (20x0=1x8=8), and together the Mesoamerican digits of "38" represent the number 68. Though the positional value of the Mesoamerican numbers is vertical rather than horizontal, so 68 would be represented as portrayed below:

Table 3: Positional value in the vigesimal system.

Base-20 exponentials	2^4	20^3	20^2	20^1	20^0
Equivalent Decimal	**160,000**	**8,000**	**400**	**20**	**1**
English words	Sets of one hundred sixty thousand	Sets of eight thousand	Sets of four hundred	Sets of twenty	ones
Sixty-eight 					

To rehearse this system, let's review and find out the numbers represented below:

A)  B)  C)  D) 

For "A", we have a five in the second column (20^1) or (20x1=20x5=100) that means five groups or sets of twenty or one hundred units. Additionally, we have digit zero in the first column (20^0), so the number for "A" is 100. "B" represents 2,000, since the digit five in the third column (20^2) we multiply a set of four hundred five times, which equals two thousand. Because in the other columns we have zeros, then the number is just 2,000. From this we can deduce that "C" is 2,003. Now, find out the value of "D".

Teaching Practices

During the opportunities of teaching and coteaching the Mesoamerican numbers, I have noticed some practices that best support student understanding of the vigesimal system and its relation to indigenous language and thought. Below, I describe two strategies that seem to fall short in promoting intercultural educations goals and mathematics and two strategies that supported greater relationships with student development of their understanding of Mesoamerican Numbers and its representations. During each strategy description, I focus on the links between language and mathematical thought by analyzing the linguistic, mathematical and symbolic dimensions to develop the meaning of numbers.

A. Translating Words and Memorize Number Names

This practice attempts to solve the problem of teaching numbers as if the learning of mathematics could be done through a dictionary translation. For example, students will receive a list of numbers with their respective words or translation, so that they can link the known number to the Ixil word for the known number. This lack of exploration of the meaning of the Ixil language other than its connection to the representation or name of a number deprives students of understanding the base-20 system. Therefore, when thinking about the Mesoamerican numbers in Ixil, they only have words that seems empty, without meaning. The process of becoming successful in mastering this knowledge would then reside on memorization without addressing the mathematical relationships of numbers in a base-20 system. As seen in Table 4, the darken cells portray the domains explore through this teaching strategy. Students might be able to retrieve the name of some numbers in Ixil, but with a limited understanding of the base twenty for how 58 would transfer mathematically into the base-20 system.

Table 4: Dimensions of the Development of Meaning of Number when Translating Words

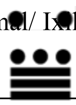
Systems\Domains	Linguistic	Mathematical	Symbolic
Decimal/ English	Fifty-eight	5 tens 8 ones	58
Vigesimal/ Ixil	Vaaxajlaval itoxk'al (eight-ten after two twenties)	2 sets of twenty 18 ones	

B. Exploring Mesoamerican Representations without Mesoamerican Language

Another strategy used to teach the Mesoamerican numbers is by focusing on the visual representations (bars, dots, shells) of these numbers. This exercise might take two routes. One way is to simply translate or find the equivalent symbol of the decimal number to the one used in the Mesoamerican numbers. This approach is similar to the prior strategy in that the new learning of the representation is isolated or disconnected from understanding the mathematical relations of numbers in a base-20 system. The other option could be the learning of how the Mesoamerican numbers are represented symbolically by understanding that the symbols represent base-20 relationship, so that the two dots in the lowest most right cell in Table 5, these can be understood as representing the value of two groups of twenty or forty. This strategy ignores the Ixil words that would show further help name and understand the mathematical pattern in the base-20 system,

Table 5: Dimensions of the Development of Meaning of Number when focusing on Representations

Systems\Domains	Linguistic	Mathematical	Symbolic
Decimal/ English	Fifty-eight	5 tens 8 ones	58
Vigesimal/ Ixil	Vaaxajlaval itoxk'al (eight-ten after two twenties)	2 sets of twenty 18 ones	



C. Uncovering Indigenous Mathematical Thought (Base-20) through Indigenous Languages (Number Names)

This strategy aims at targeting the three domains for the development of meaning of number. After the exploration of the Mesoamerican number representations and how they portray base-20 relations as described in B. Here, students go further by analyzing a list of numbers with the equivalent Ixil from 1-100 or more (see Appendix A), so that students can find how patterns in the naming of the Mesoamerican numbers in Ixil is linked to mathematical patterns in base-20 systems. When students work through this approach, indigenous language and mathematics indigenous become more explicit.

Table 6: Dimensions of the Development of Meaning of Number when focusing on Indigenous Thought and Language

Systems\Domains	Linguistic	Mathematical	Symbolic
Decimal/ English	Fifty-eight	5 tens 8 ones	58
Vigesimal/ Ixil 	Vaaxajlaval itoxk'al (eight-ten after two twenties)	2 sets of twenty 18 ones	

D. Contrasting Numerical Representations with the Meaning of the Numbers across Number Systems and Languages

The last strategy attempts to include a more comprehensive development of students understanding of base-20 system and Ixil by contrasting it with base-10 patterns. The language and well as the mathematical relations and patterns are explored in each language and system. As Table 7 shows the three domains of the development of the meaning of number are addressed through this strategy.

Table 7: Dimensions of the Development of Meaning of Number when Contrasting across Languages and Number Systems.

Systems\Domains	Linguistic	Mathematical	Symbolic
Decimal/ English	Fifty-eight	5 tens 8 ones	58
Vigesimal/ Ixil 	Vaaxajlaval itoxk'al (eight-ten after two twenties)	2 sets of twenty 18 ones	

Discussion

Greater understanding of one number system becomes more explicit through the process of leaning, exploring, or develop understanding of another number system. The process of understanding how patterns in number systems are established eases the learning and creation of new numerical systems. Mathematics education can empower intellectually, pragmatically and socio-politically (Skovsmose, 2011). The learning of Mesoamerican mathematics nurtures the learning and promotion of indigenous thought and culture (Grisby, 2004). As presented above, language has a central role in these processes of learning and empowerment. Language is rooted in the use and understanding of the word. "The meaning of the word is a phenomenon of the thought only in the way that the thought is related with the word, and is incarnated in it, and vice-versa" (Vigotski, 2012, p. 436). Since, the relationship of understanding indigenous mathematical thought and epistemologies resides in how our thought fuses with the indigenous language through the meaning of the mathematical words, numbers and relations. Because "thought is not reflected in the word, but it is realized within it" (p. 438). However, the strong influence of the decimal number system in the way we think mathematically is undermining other ways of thinking mathematically that the Mesoamerican mathematics and others provide. The learning of these mathematics supports not only the promotion and development of indigenous thought, but of the diversity of human heritage.

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Appendix A:

Selected Sample of Names of Numbers in Ixil

1 Ma'l
2 Ka'va'l
3 Oxva'l
4 Kaava'l
5 Ova'l
6 Vaajil
7 Jujva'l
8 Vaaxajil
9 Beluval
10 Laval
11 Junlaval
12 Kab'laval
13 Oxlaval
14 Kaalaval
15 O'lavaj
16 Vajlaval
17 Jujlaval
18 Vaaxajlaval
19 Belelaval
20 Vingil
21 Ma'lika'k'al
22 Ka'va'l ika'k'al
23 Oxval ika'k'al
24 Kaava'l ika'k'al
25 O'va'l ika'k'al
26 Vaajil ika'k'al
27 Jujval ika'k'al
28 Vaaxajil ika'k'al
29 B'eluval ika'k'al
30 Laval ika'k'al
31 Junlaval ika'k'al
32 Kab'laval ika'k'al
33 Oxlaval ika'k'al
34 Kaalaval ika'k'al
35 Olavaj ika'k'al
36 Vajlaval ika'k'al
37 Jujlaval ika'k'al
38 Vaaxajlaval ika'k'al
39 Belelav ika'k'al
40 Ka'k'al
41 Ma'l itoxk'al
42 Ka'va'l itoxk'al
43 Oxv'al itoxk'al
44 Kaava'l itoxk'al
45 Oval itoxk'al
46 Vaajil itoxk'al
47 Jujva'l itoxk'al
48 Vaajil itoxk'al
49 Beluval itoxk'al
50 Laval itoxk'al
51 Junlaval itoxk'al
52 Kab'laval itoxk'al
53 Oxlaval itoxk'al
54 Kaalaval itoxk'al
55 O'laval itoxk'al
56 Vajlaval itoxk'al
57 Jujlaval itoxk'al
58 Vaaxajlaval itoxk'al
59 Belelaval itoxk'al
60 Oxk'alal
61 Ma'l ikaak'al
62 Ka'va'l ikaak'al
63 Oxval ikaak'al

70 Laval ikaak'al

The voices of teachers teaching in a very remote school provide insight into teachersâ€™ responses to the mismatch between the system expectations and the teaching context. Teacher interviews in a small Northern Territory school, conducted within an ethnographic study, showed that teachersâ€™ decisions regarding the level of mathematics curriculum taught were informed by studentsâ€™ prior learning and by the language dynamic in their classrooms.Â Mathematical enculturation: A cultural perspective on mathematics education. Dordrecht: Kluwer. Christie, M. (2007).Â Challenges for teacher education: The mismatch between beliefs and practice in remote Indigenous contexts.Â Closing the policyâ€"practice gap: Making Indigenous language policy more than empty rhetoric. "This book is for every math teacher who has ever been frustrated and confused about why many math students â€²just donâ€²t get itâ€².Â Teaching Numeracy is filled with ideas and strategies that you can use in the classroom no matter what your personal style of teaching is. Everything is written in a format that makes sense, and provides a nice template for assembling a lesson plan using best practices techniques! Read more.