Logic and Abduction

Cognitive Externalizations in Demonstrative Environments

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Abstract: In her book *Abductive Reasoning* Atocha Aliseda (2006) stresses the attention to the logical models of abduction, centering on the semantic tableaux as a method for extending and improving both the whole cognitive/philosophical view on it and on other more restricted logical approaches. I will provide further insight on two aspects. The first is related to the importance of increasing logical knowledge on abduction: Aliseda clearly shows how the logical study on abduction in turn helps us to extend and modernize the classical and received idea of *logic*. The second refers to some ideas coming from the so-called *distributed cognition* and concerns the role of logical models as forms of cognitive externalizations of preexistent in-formal human reasoning performances. The logical externalization in objective systems, communicable and sharable, is able to grant stable perspectives endowed with symbolic, abstract, and rigorous cognitive features. I will also emphasize that Aliseda especially stresses that this character of stability and objectivity of logical achievements are not usually present in models of abduction that are merely cognitive and epistemological, and of extreme importance from the computational point of view.

1. INTRODUCTION

In her book *Abductive Reasoning* Atocha Aliseda (2006) stresses the attention to the logical models of abduction, centering on the semantic tableaux as a method for extending and improving both the whole cognitive/philosophical view on abduction and on other more restricted logical approaches. I think the book wonderfully achieves many results. Logic is definitely offered to the appreciation of an interdisciplinary cognitive audience in both its plasticity and rigor in modeling various kinds of reasoning, beyond the rigid character of many of the traditional perspectives.

I would like to provide further insight on two aspects. The first is related to the importance of increasing logical knowledge on abduction: Aliseda clearly shows how the logical study on abduction in turn helps us to extend and modernize the classical and received idea of *logic*. The second refers to some issues coming from the so-called *distributed cognition* approach and concerns the role of logical models as forms of cognitive externalizations of preexistent in-formal human reasoning performances. The logical externalization in objective systems, communicable and sharable, is able to grant stable perspectives endowed with symbolic, abstract, and rigorous cognitive features. Indeed, Aliseda says, this character of stability and objectivity of the logical achievements is not usually present in models of abduction that are merely cognitive and epistemological and, moreover, they are central to computational implementation.

As I described about six years ago in my book *Abduction, Reason, and Science* (Magnani, 2001) Peirce clearly indicated the importance of logic (first order syllogism) to grasp the inferential status of abduction, also creating a wonderful new broad semiotic view at the same accompanied by the well-known philosophical commitment to the new vision on pragmatism.[1] Given the restricted scope - classical, in terms of first order syllogisms - of the logical tools that were available to him the logical framework just depicted abduction as the well-known “fallacy of affirming the consequence”. Aliseda’s book belongs to this fundamental Peircian “logical” tradition: it presents a plenty of recent logical models of abduction which are clearly illustrated in their rigorous “demonstrative” frameworks.

[1] Aliseda (2006) clearly shows in chapter seven how abduction is at the basis of Peirce’s pragmatism.

2. MODEL-BASED REASONING IN DEMONSTRATIVE FRAMEWORKS

It is well-known that the kind of reasoned inference that is involved in selective and creative abduction[2] goes beyond the mere relationship that there is between premises and conclusions in valid "classical" deductions, where the truth of the premises guarantees the truth of the conclusions, but also beyond the relationship that there is in probabilistic reasoning, which renders the conclusion just more or less probable.

On the contrary, we can see selective and creative abduction as formed by the application of "heuristic procedures" that involve all kinds of good and bad inferential actions, and not only the mechanical application of rules. It is only by means of these heuristic procedures that the acquisition of *new* truths is guaranteed. Also Peirce's mature view on creative abduction as a kind of "inference" seems to stress the strategic component of reasoning.

Many researchers in the field of philosophy, logic, and cognitive science have sustained that also deductive reasoning consists in the employment of logical rules in a heuristic manner, even maintaining the truth preserving character: the application of the rules is organized in a way that is able to recommend a particular course of actions instead of another one. Moreover, very often the heuristic procedures of deductive reasoning are in turn performed by means of an "in-formal" *model-based* abduction.[3] So humans apply rudimentary abductive/strategic ways of reasoning in deduction too. A most common example of strategic process that leads to the formation of new hypotheses (creative abduction) is the usual experience people have of solving problems in geometry in a model-based way trying to devise proofs using diagrams and illustrations: of course the attribute of creativity we give the abduction in this case does not mean that it has never been made before by anyone or that it is original in the history of some knowledge (in this particular case the ancient Greek geometers were the "real" creative abducers!).

Hence, we have to say that a kind of "in-formal" model-based abductions also operate in deductive reasoning performed by human who use logical systems. Following Hintikka and Remes's analysis (1974) proofs of general

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[2] Epistemologically selective abduction occurs when we reach a hypothesis among – to use a word of the logical tradition also exploited by Aliseda – already available "abducibles" hypotheses (like for instance in the case of medical diagnosis). Creative abduction occurs when, through our reasoning processes, we are able to create "new" abducibles, which can be tested and added to [or which can replace] the available ones (Magnani, 2001).

[3] In general model-based abduction takes advantage of internal (or of external models suitably re-internalized) that are not merely symbolic/propositional but which exploit for example diagrams, visualization, configurations, schemes, thought experiments, and so on (Magnani, 2001).

implication in first order logic need the use of instantiation rules by which "new" individuals are introduced, so they are "ampliative". In ordinary geometrical proofs auxiliary constructions are present in term of "conveniently chosen" figures and diagrams. In Beth's method of *semantic tableaux* the "strategic ability" performed by humans to construct impossible configurations is undeniable (Hintikka, 1998; Niiniluoto, 1999). Also Aliseda provides interesting uses of the semantic tableaux as constructive representations of theories, where for example, abductive expansions and revisions, derived from the belief revision framework, operate over them. In the case of tableaux, their symbolic character is certainly fundamental, but it is also particularly clear they also are configurations – model-based – of proofs externalized through suitable notations.[4]

Following Hintikka, we can say that the "ground floor" of deductive reasoning, the first-order logic, is nothing but operating with certain models or approximations of models, as is just simply demonstrated by some fundamental techniques such as Beth's semantic tableaux. It is important to note that Hintikka is perfectly aware of the double character of these "models", *internal* and/or *external*:

> These models can be thought of as being mental, or they can be taken to consist of sets of formulas on paper – or in this day and age perhaps rather on the screen and in the memory of a computer. In fact, from this perspective all rules of "logical inference" obviously involve "mental models". Johnson-Laird's discovery hence does not ultimately pertain to the psychology of logic. It pertains, however confusedly, to the nature of logic itself. *The most basic deductive logic is nothing but experimental model construction* (Hintikka, 1997, pp. 69-70).

In this way Hintikka rejoins the distributed cognition approach to logic I have also stressed in Magnani (2005), where the interplay between internal and external (as kinds of "semiotic anchors" – *symbolic*, in this case, Magnani 2006b) aspects of logical reasoning are illustrated. For example the role in logical deduction of the strategies of experimental (counter) model-construction is stressed, but also the importance of the introduction of the right new individuals by means of existential instantiation to be introduced in the model. The most important "strategic" question – in deductive reasoning – is to determine in what order the instantiations are to be treated. In geometrical reasoning the role of existential instantiation is obvious and occurs through the iconic so-called "auxiliary constructions", where

[4] It is worth to be noted that semantic tableaux method provides further insight on the problem of theory evaluation, intrinsic in abductive reasoning. In chapters six and eight, Aliseda shows how semantic tableaux can deal with "causal" aspects of abductive reasoning that cannot be considered with the only help of the logic programming tradition.

conceptually manipulating a configuration of geometrical objects and extending it by introducing new individuals is at stake. The possible creative character is for example reflected in the fact that there is not always a mechanical (recursive) method for modeling these human deductive performances. Of course, as Aliseda shows in chapter four “Abduction as computation”, a suitable computational counterpart can take advantage of algorithms which render mechanical the suitably chosen reasoning processes, and so suitable to be implemented in a computational program.

The logical tradition of Frege and Russell rejected all reasoning that had been made in terms of geometrical icons as being responsible for introducing an appeal to intuition. On the contrary, I am very inclined to agree with Hintikka, who maintains that the traditional idea of logical reasoning as a discursive process is wrong, it is an “optical illusion”, because all deduction is a form of “experimental model construction” that follows that interplay between internal and external representations I have already indicated. It is important instead to note that for instance already at the level of elementary geometry:

> […] geometrical figures are best thought of as a fragmentary notation for geometrical proofs alternative to, but not necessarily intrinsically inferior to, the “purely logical” notation of formalized first order logic. […] They are intrinsic features of certain deductive methods. They are part of the semantics of logical reasoning, not only of its psychology or its heuristics. If it is suggested that heuristic ways of thinking are needed to make mathematical reasoning intuitive, I will borrow a line from Wittgenstein’s *Tractatus* 6.233 and say that in this case the language (notation) itself provides the intuitions (Hintikka, 1997, p. 73).

Moreover, in the case of human performances, in many forms of deductive reasoning there are not trivial and mechanical methods of making inferences but we have to use “models” and “heuristic procedures” that refer to a whole set of strategic principles. All the more reason that Bringsjord (1998) stresses his attention on the role played by a kind of “model based deduction” that is “part and parcel” of our establishing Gödel’s first incompleteness theorem, showing the model-based character of this great abductive achievement of formal thought.[5]

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[5] Many interesting relationships between model-based reasoning in creative settings and the related possible deductive “dynamic” logical models are analyzed in Meheus (1999) and Meheus and Batens (2006). Dynamic logic is also related to the formal treatment of inconsistencies.

3. EXTERNAL AND INTERNAL REPRESENTATIONS

3.1 Logic Programs as Agents: External Observations and Internal Knowledge Assimilation

It is in the area of *distributed cognition* that the importance of the interplay between internal and external representations has recently acquired importance (cf. for example Clark, 2003, and Hutchins, 1995). This perspective is particularly coherent with the so-called agent–based framework (Magnani, 2006a). It is interesting to note that a clear attention to the agent–based nature of cognition and to its interplay between internal and external aspects can also be found in the area of logic programming, which Aliseda describes as one of the two main ways – we already said that the other is the semantic tableaux method – of logically and computationally deal with abduction.

I think in logic programming a new idea of logic – contrasted with the classical one – arises, which certainly opens to abduction the door that grants access to its full treatment through logical systems. Indeed, logic programs can be seen in an agent-centered, computationally-oriented and purely syntactic perspective. Already in 1994 Kowalski (1994) in "Logic without model theory" introduced a knowledge assimilation framework for rational abductive agents, to deal with incomplete information and limited computational capacity.

"Knowledge assimilation" is the assimilation of new information into a knowledge base, "as an alternative understanding of the way in which a knowledge base formulated in logic relates to externally generated input sentences that describe experience". The new pragmatic approach is based on a proof-theoretic assimilation of observational sentences into a knowledge base of sentences formulated in a language such as CL.[6] Kowalski proposes a pragmatic alternative view that contrasts with the model-theoretic approach to logic. In model theory notions such as *interpretation* and *semantic structures* dominate and are informed by the philosophical assumption that experience is caused by an independent existing "reality composed of individuals, functions and relations, separate from the syntax of language"

On the contrary logic programs can be seen as agents endowed with deductive databases considered – Kowalski says – as "theory presentations" from which logical consequences are derived, both in order to *internally*

[6] CL, computational logic, refers to the computational approach to logic that has proved to be fruitful for creating non–trivial applications in computing, artificial intelligence, and law.

solve problems with the help of *theoretical sentences* and in order to assimilate new information from the *external* world of observations (*observational sentences*). The part of the knowledge base, which includes observational sentences and the theoretical sentences that are used to derive conclusions that can be compared with observations sentences, is called *world model*, considered a completely syntactic concept: "World models are tested by comparing the conclusions that can be derived from them with other sentences that record inputs, which are observational sentences extracted – *assimilated* – from experience". The agent might generate outputs – that are generated by some plan formation process in the context of the agent's "resident goals" – which affect its environment and which of course can affect its own and other agents' future inputs. Kowalski concludes "The agent will record the output, predict its expected effect on the environment using the 'world model' and compare its expectations against its later observations".

I think the epistemological consequence of this approach is fundamental, also to understand the philosophical perspective of Aliseda's book: in model theory truth is a static correspondence between sentences and a given state of the world. In Kowalski's computational and "pragmatic" theory, the important is not the correspondence between language and experience, but the appropriate assimilation of an inevitable and continuous flowing input stream of "external" observational sentences into an ever changing "internal" knowledge base (of course the fact that computational resources available are bounded suggests to the agent to make the best use of them, for instance avoiding redundant and irrelevant derivation of consequences). The correspondence (we can say the "mirroring") between an input sentence and a sentence that can be derived from the knowledge base is considered by Kowalski only a limiting case. Of course the agent might also generate its own hypothetical inputs, as in the case of abduction, induction, and theory formation.

Aliseda seems to acknowledge this fact and further improves this perspective. The task is accomplished with the help of the semantic tableaux framework which can control in several ways various and meaningful logical and computational abductive strategies, some of them reflecting types of abducing already present in "actual" human performances: "That is, we must provide the automatic procedures to operate a logic, its control strategy, and its procedures to acquire new information without disturbing its coherence and hopefully achieve some learning in the end" (2006, p. 23). It is important to stress that thanks to the new Aliseda's book the fact that in the logic of abduction the sensitivity to the growth of information [and the suitable extension of logical language] is definitely considered fundamental for the whole logic itself. Her analogy with the non-Euclidean revolution is striking and conclusive: "Whether non classical modes of reasoning are really logical is like asking if non-Euclidean geometries are really geometries" (2006, p. 92). We know that that discovery represented an irreversible extension of geometry and mathematics beyond the restricted "intuitive" areas of the elementary perspectives.

The conceptual framework above, that is derived from a computationally–oriented logic approach that strongly contrasts with the traditional one in terms of model theory, is extremely interesting. It stresses the attention on the flowing interplay between internal and external representations/statements, so *epistemologically* establishing the importance of the agent–based character of cognition and so of "logical" cognition. In the recent cognitive science approach in terms of "distributed cognition" this perspective is convenient also for depicting actual human beings' cognition so far as we are interested in studying its essential distributed dynamics.

3.2 Distributed Cognition in Organic Agents: External and Internal Representations

Mind is limited, both from a computational and an informational point of view: the act of delegating some aspects of cognition becomes necessary. It is in this sense that we can say that cognition is essentially *multimodal*.[7] In addition, we can say that, adopting this perspective, we can give an account of the complexity of the whole human cognitive systems as the result of a complex interplay and *coevolution* of states of mind, body, and external environments suitably endowed with cognitive (in the cases illustrated in this commentary "logical") significance. The "agent-based" view I have illustrated in the previous subsection aims at analyzing the features of "real" human thinking agents and "ideal" logical agents by recognizing the fact that a being-like-us agent functions "at two levels" and "in two ways". I define the two levels as *explicit* and *implicit* thinking. *Agent-based* perspective in logic has the power of recognizing the importance of both levels.

We maintain that representations are external and internal. We can say that

[7] Thagard (2005, 2006) observes, that abductive inference can be visual as well as verbal, and consequently acknowledges the sentential, model–based, and manipulative nature of abduction I have illustrated in my book (Magnani, 2001). Moreover, both data and hypotheses can be visually represented: "For example, when I see a scratch along the side of my car, I can generate the mental image of grocery cart sliding into the car and producing the scratch. In this case both the target (the scratch) and the hypothesis (the collision) are visually represented. [...] It is an interesting question whether hypotheses can be represented using all sensory modalities. For vision the answer is obvious, as images and diagrams can clearly be used to represent events and structures that have causal effects (2006)." Indeed hypotheses can be also represented using other sensory modalities.

- *external representations* are formed by external materials that re-express (through reification) concepts and problems that are already present in the mind or concepts and problems that do not have a *natural home* in the brain;
- -*internalized representations* are internal re-projections, a kind of recapitulations, (learning) of external representations in terms of neural patterns of activation in the brain. They can sometimes be "internally" manipulated like external objects and can originate new internal reconstructed representations through the neural activity of *transformation* and *integration*.

3.3 Internal, External, and Hybrid Inducers and Abducers: External Semiotic Anchors

In the following I will illustrate some features of this extraordinary interplay between human brains and the *ideal cognitive systems* they make, and so ideal logical (and computational) agents. We acknowledge that material artifacts like for example *inductive and abductive logical and computational agents* are tools for thoughts as is language: tools for exploring, expanding, and manipulating our own minds. A novel perspective on external ideal logical agents can be envisaged.

Human beings spontaneously (and also animals, like already Peirce maintained) perform more or less rudimentary abductive and inductive reasoning. Starting from the low–level "in-formal" inferential performances like for example hasty generalization or simple abductive diagnoses, widespread in children and adult humans, that certainly can represent a strategic success (for instance survival) and a cognitive failure (they are not truth-preserving, and so epistemologically unsatisfactory) human beings arrived to the externalization of "theoretical" inductive and abductive agents as *ideal agents*, logical and computational, like the ones Aliseda illustrates in her book. It is in this way that *merely successful strategies* are replaced with *successful strategies* that also tell the "more precise truth" about things. These external representations can be usefully re-represented in our brains (if this is useful, simple, and possible), and they can originate new improved organic (mentally internal) ways of inferring or suitably exploited in a hybrid manipulative interplay, as I have already said above. In summary, we can copy ("recapitulating" them through internalization) ways of reasoning (or fragments of them) from some aspects of what we have externalized over there, in the external environment, for instance in "ideal" logical systems/agents.

From this perspective human beings are hardwired for *survival* and for *truth* alike so best inductive and abductive strategies can be built and made explicit, through self-correction and re-consideration (a process that is at play at least since for example the time of the "ideal" inductive Mill's methods). Furthermore, human beings are agents that can cognitively behave as *hybrid agents* that exploit in reasoning both internal representations and externalized logical representations and tools, but also the mixture of the two.

Let 's consider the example of the externalization of some inferential skills in logical demonstrative systems, like for example the ones that are at the basis of logic programming and semantic tableaux, both illustrated and improved by Aliseda. They present interesting cognitive features (cf. also Longo, 2005) which I believe deserve to be further analyzed:

1. *symbolic*: they activate and semiotically "anchor" meanings in material communicative and intersubjective *mediators* in the framework of the phylogenetic, ontogenetic, and cultural reality of the human being and its language. I have already said it can be hypothesized these logical agents originated in embodied cognition, gestures, and manipulations of the environment we share with some mammals but also non mammal animals (cf. the case of the complicated monkeys' knots, endowed with implicit mathematical features, and pigeons' categorization, as implicit concept formation, in Grialou, Longo, and Okada, 2005).
2. *abstract*: they are based on a *maximal independence* regarding sensory modality; they strongly stabilize experience and common categorization. The maximality is especially important: it refers to their practical and historical invariance and stability;
3. *rigorous*: the rigor of proofs is reached through a difficult practical experience. For instance, in the case of mathematics and logic, as the maximal place for "convincing and sharable" reasoning. Rigor lies in the stability of proofs and in the fact they can be iterated. Following this perspective mathematics is the best example of maximal stability and conceptual invariance. Logic is in turn a set of proof invariants, a set of structures that are preserved from one proof to another or which are preserved by proof transformations. As the externalization and result of a distilled praxis, the praxis of proof, it is made of maximally stable regularities;
4. I also say that a *maximization of memorylessness*[8] "variably" characterizes demonstrative reasoning. This is particularly tangible in the case of the vast idealization of classical logic and

[8] I derive this expression from Leyton (2001) that introduces a very interesting new geometry where forms are no longer memoryless like in classical approaches such as the Euclidean and the Kleinian in terms of groups of transformations

> related approaches. The inferences described by classical logic do not yield sensitive information – so to say – about their real past life in human agents' use, contrarily to the "conceptual" – narrative – descriptions of human informal non-demonstrative processes, which variously involve "historical", "contextual", and "heuristic" memories. Indeed many informal thinking behaviors in human agents – for examples abductive inferences, especially in their generative part – are context-dependent. As already noted their *stories* vary with the multiple propositional relations the human agent finds in his/her environment and which he/she is able to take into account, and with various cognitive reasons to change his/her mind or to think in a different way, and with multiple motivations to deploy various tactics of argument.

In this perspective Gabbay and Woods say:

> Good reasoning is always good in relation to a goal or an agenda which may be tacit. [. . .] Reasoning validly is never itself a goal of good reasoning; otherwise one could always achieve it simply by repeating a premiss as conclusion, or by entering a new premiss that contradicts one already present. [. . .] It is that the reasoning actually performed by individual agents is sufficiently reliable not to kill them. It is reasoning that precludes neither security not prosperity. This is a fact of fundamental importance. It helps establish the fallibilist position that it is not unreasonable to pursue modes of reasoning that are known to be imperfect (Gabbay and Woods, 2005, pp. 19-20].

Human agents, as practical agents, are *hasty inducers* and *abducers* and bad predictors, unlike ideal (logical and computational) agents. In conclusion, we can say informal abductive inferences in human agents have a memory, a story: consequently, an *abductive ideal logical agent* which formalizes those human skills has to variably weaken many of the aspects of classical logic and to overcome the relative demonstrative limitations.

I think that a great contribution given to logic by Aliseda is the improvement of the *semantic tableaux method* (and its application to the logic of abduction and to the enhancement of other logical models of abduction, like for example the belief-revision framework). The new extended semantic tableaux method fulfills the request of "weakening" the rigidity of classical logic but also of many non standard logics strictly related to it, helping to further open the new era of logic: the attention to the role of *meta-levels* – for instance in the logic of abduction – formalizes the flexibility and "historicity" of many kinds of human thinking which are meaningful in certain application areas they address.

Aliseda's conclusion is clear, and by implicitly reflecting the four aspects I have just illustrated, also leads not only to a new perspective on abduction but also to a new conception of logic:

> The various types of abductive explanatory styles in a larger universe of other deductive and inductive systems of logic naturally commit us to a global view of logic […] in which there is a variety of logical systems which rather than competing and being rival to each other, they are complementary in that each of them has a specific notion of validity corresponding to an extra-systemic one and a rigorous way for validating arguments, for it makes sense to speak of a logical system as correct or incorrect, having several of them. And finally, the global view states for abduction that it must aspire to global application, irrespective of subject matter, and thus found in scientific reasoning and in common sense reasoning alike (2006, p. 89).

We can conclude by stressing the fact that human informal non-demonstrative inferential process of abduction (and of induction) is more and more externalized and objectified – and Aliseda contributes to this extension - at least in three ways:

1. through Turing's Universal Practical Computing Machines we can have running programs – often based on logic, that are able to mimic – and enhance – "the actions of a human computer very closely" (Turing, 1950), and so - amazingly - also those human agents' "actions" that correspond to the complicated inferential performances like abduction (cf. the whole area of artificial intelligence);
2. human non-demonstrative processes are more and more externalized and made available in forms of explicit narratives and learnable templates of behavior (cf. also the study of fallacies as important tools of that human "kit" that provides evolutionary advantages, in this sense any fallacy of the affirming the consequent – which depicts abduction in classical logic – "can" be better than nothing (Woods, 2004).[9]
3. new "demonstrative" systems – ideal logical agents – are created able to model and make rigorous in a *deductive* way many non-demonstrative thinking processes, like abduction, analogy, creativity, spatial and visual reasoning, etc.

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[9] Cf. also Gabbay and Woods (2005, pp. 33-36).

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