the role of the Jews in mathematical scholarship during the Middle Ages (neglected in all standard American textbooks except that of Victor Katz), the growth of national schools of mathematics, and specifically mathematical institutions, such as mathematical societies and journals.

There is not a dull essay in this collection. In a fine example of European unity Colin Fletcher writes in French about the community around Mersenne (and fleshes out the usual generalities with vivid specifics) and Marie-José Durand-Richard writes about the British school of algebra. Of the 23 essays 12 are in French and 11 in English. The introduction and conclusion are written in both languages. Americans who have become wary of "eurocentrism" may be disturbed by the title. As the volume under review shows, however, the very attempt to study European mathematics accurately provides the best antidote for any such cultural narrowness. Only within the broader context of mathematics in general can such a thing as European mathematics even be defined. A collection of essays on the history of mathematics more interesting and informative than this book is difficult to imagine.

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Poincaré and the Three-Body Problem. By June Barrow-Green. Providence, RI (American Mathematical Society). 1997. xv + 272 pp. ISBN 0-8218-0367-0.

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On 21 January 1889, Henri Poincaré was awarded the Prize of King Oscar II of Sweden and Norway for the memoir entitled *Sur le problème des trois corps et les équations de la dynamique*, which made an important contribution to the progress of mathematics in general and to the understanding of the three-body problem in particular. This caused not only international recognition of Poincaré's work and of the new research directions he had opened but also a scandal that at some point threatened to spread beyond the boundaries of the mathematical community. Only the tactful intervention of Gösta Mittag-Leffler, the organizer of the competition and an advisor to the King, saved the reputation of the prize and of those involved in awarding it.

The scandal began with an error that Poincaré discovered in the prize paper while preparing the manuscript for publication and that he corrected during the following months. The discovery of the error, however, came too late to stop the printing of the latest issue of *Acta Mathematica*, in which the flawed memoir appeared. Some copies even reached their subscribers. As editor-in-chief of *Acta*, Mittag-Leffler decided to destroy the whole production and reprint the issue. The substantial costs, which exceeded threefold the awarded sum, were entirely covered by Poincaré. Given the circumstances, several people questioned the fairness of the competition. Hugo Gyldén, a Swedish mathematical astronomer, claimed that he deserved the prize since he had found a power series solution of the three-body problem, convergent for all time, thus answering the question that

Weierstrass had posed for the competition. Others argued that Poincaré had contributed much more than simply addressing one of the jury's problems. Today, we know that the latter were right and that Gyldén's solution was not convergent in the mathematical sense.

Poincaré's new result, obtained under so much psychological pressure, was the first glimpse of chaotic behavior in a dynamical system, a fact fully understood by the mathematical community only three quarters of a century later, after Birkhoff and then Smale polished it and pointed out its importance. The French mathematician had been much ahead of his time. What most of his contemporaries saw was not the achievement but the mistake and the scandal surrounding it. The patina of time, however, covered the petty details, and until recently the historical facts had been virtually forgotten. Those few who knew something discussed it only in private.

But in the late 1980s these private discussions intensified and the subject suddenly received the attention of several people, who started researching it independently. The first account appeared in 1993 [7] in a book written by the Canadian mathematics journalist, Ivars Peterson. In the same year, the second account was published by Daniel Goroff of Harvard University in the introduction to his English translation of *Les méthodes nouvelles de la mécanique céleste* [6]. The third and fourth appeared one year later in the same issue of the *Archive for History of Exact Sciences*, one written by the British historian of mathematics June Barrow-Green [2]—the published version of her doctoral thesis—and the other by the Swedish mathematician, Karl Andersson [1]. The fifth account was published in 1996 in a book coauthored by the present reviewer with Philip Holmes of Princeton University [5], and the last—the work under review and a development of the aforementioned dissertation—appeared in 1997 [3].

Barrow-Green's book deals with the work of Poincaré on the three-body problem, which spanned the years from 1879, when he completed his thesis, to 1912, the year of his death. As the mathematical encyclopedist of his time and a highly regarded physicist and philosopher, Poincaré studied many questions and his interests changed continuously. But there was a perennial love, almost an obsession: the system of differential equations describing the three-body problem, which he pursued during his entire career, to which he returned time and again, and which motivated an important part of his other work. Therefore writing a book on Poincaré's contribution to this subject is a difficult task that requires not only mathematical and historical erudition, but an intimate acquaintance with Poincaré's entire intellectual universe. Barrow-Green knows this, and still takes the challenge. I was pleased to follow her journey over the many inevitable obstacles.

The book is centered around King Oscar's competition and Poincaré's memoir, the evolutions of which are analyzed in detail. The author has a good story to tell and makes out of it not only a sound historical account but exciting, smooth, and attractive reading. After a brief introduction, she defines the three-body problem and outlines its history, mentioning some of the main contributions made by Newton, Johann and Daniel Bernoulli, Euler, Lagrange, Laplace, Clairaut, Delaunay, Gyldén, Lindstedt, and Hill. Then she describes Poincaré's work before 1889, emphasizing his results in celestial mechanics. Chapters 4, 5, and 6, which form the core of the book, recount the history of King Oscar's competition, make a detailed analysis of the two printed versions of the memoir, try to reconstitute their original source sent into the competition, and discuss the reaction of peers like Moulton, Gyldén, Minkowski, Hill, Whittaker, and Kelvin to Poincaré's memoir. The author then analyzes Poincaré's related work after 1889: his masterpiece *Les méthodes nouvelles*, its didactic counterpart *Leçons de mécanique céleste*, some other research papers on the three-body problem, and the famous last geometric theorem, which led Birkhoff to the fixed-point result that bears his name. This is followed by a nice description of the associated and subsequent work of other mathematicians such as Lyapunov on stability; Painlevé, von Zeipel, Levi-Civita, Chazy, McGehee, Saari, and Xia on singularities; Darwin, Moulton, and Strömgren on quantitative results; and Bisconcini and especially Sundman on the power series solution of the three-body problem. Unfortunately, the work of Quidong Wang (see [4]), who in 1991 provided a global solution of the n-body problem, is not mentioned. The author then dedicates a separate chapter to the related work of Poincaré's main followers, Hadamard and Birkhoff, and ends with a short discussion of the Kolmogorov–Arnold–Moser theory, which answers Weierstrass's prize question.

Undoubtedly, from now on this book will be the main historical source for the three-body problem of celestial mechanics and a landmark reference for Poincaré's work. But while with respect to the first aspect, which was the author's reason for engaging in this enterprise, Barrow-Green's contribution is outstanding, the second aspect is less prominent. Though the author usually succeeds in following Poincaré's ideas and their subsequent development, she sometimes fails to see connections not only with the work of other authors but also within Poincaré's own work. For example, on p. 33, at the end of the second paragraph, Barrow-Green describes an early result of Poincaré concerning the asymptotic behavior of planar flows, but fails to bridge it to the work of Bendixson and the Poincaré-Bendixson theorem mentioned in the last paragraph of p. 176. Another example, from within Poincaré's work this time, is that of missing the connection between two papers in which the idea of *transverse section*—and implicitly that of *first return map*—appears. On p. 38 the author recognizes correctly that the *transverse section* has been "fundamental for the future of dynamical systems theory," but fails to see that Poincaré had earlier used it when proving a result that she comments on in the third and fourth paragraphs of p. 33.

It is laudable that the author has taken on the difficult task of providing her own translations. At some points, however, she misses the right interpretation. For example, the notion of *invariant intégral*, discussed in detail (pp. 39, 83–88, 160, 177) and recognized to be "another new and important idea...which was...to play a fundamental role" in the prized memoir (p. 39), is translated as *invariant integral* (instead of *integral invariant*, as it correctly appears in [6]). This is incorrect not only grammatically (in French, unlike English, the adjective follows the noun) but essentially too: the main feature that the notion captures is that of an invariant quantity, expressed by an integral, not that of an integral which happens to be invariant.

But small slips such as these lack the power to diminish the value of Barrow-Green's work. They rather look like wrinkles that give a human expression to a candid face. I enjoyed reading *Poincaré and the Three-Body Problem*, and I warmly recommend it to anyone interested in the history of mathematics in general and of celestial mechanics in particular not only for its merit as a sound historical account but also for sheer pleasure.

Study it consulting the bibliography or browse it while tanning on the beach. You will like it either way.

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The Quest for Longitude: The Proceedings of the Longitude Symposium, Harvard University, Cambridge, Massachusetts, November 4–6, 1993. Edited by William J. H. Andrewes. Cambridge, MA (Collection of Historical Scientific Instruments). 1996. 437 pp. $75.00. ISBN 0-9644329-0-0.

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The perfection of time measurement in the 18th century was motivated by the need to determine longitude at sea. In meeting the challenge of producing a chronometer for use at sea, John Harrison (1693–1776) constructed the most accurate timepiece of his age, indeed in some ways the finest yet constructed. Trained as a carpenter and without formal education, he succeeded in this despite the resistance of a political–scientific establishment that favored astronomical methods for longitude determination.

Everyone accepts that there are mathematical principles embodied in clocks and navigational instruments, but, as with medieval cathedrals, just how much mathematics is effectively required or consciously used by a master builder or craftsman, even as late as the 18th century, seems not very evident. As far as clock making goes, the answer given by this collection of papers selected from the 1993 longitude symposium at Harvard University is that mathematics had little or no involvement. Under mathematics in the index there are no entries after p. 65. The mathematician and historian of mathematics, Poincare and the Three Bo has been added to your Cart. Add to Cart. Turn on 1-Click ordering for this browser.Â June Barrow-Green (Author). â€º Visit Amazon's June Barrow-Green Page. Find all the books, read about the author, and more. See search results for this author. Are you an author? Learn about Author Central. June Barrow-Green (Author). 2.2 out of 5 stars 2 customer reviews. See all formats and editions Hide other formats and editions.Â This book is somewhere between an historical essay and a blow-by-blow analysis of one of Poincare's most important papers, which gave rise to many significant developments in 20th-century mathematics. You have to really like the subject of differential equations to get into this. Read more. In the first three stages, the circular restricted three-body problem is considered, and the trajectory is designed by using invariant manifolds and the differential correction method. The simulation results show that the transfer trajectory designed by means of the invariant manifolds of the Lyapunov orbit costs lower energy and shorter time of flight than that designed by means of the invariant manifold of the Halo orbit. In the fourth stage, the two-body problem is considered, and the aerobraking method is applied. A comparative performance analysis of static and rotating atmospheric models