Symmetries of equations of quantum mechanics

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Abstract

This book is devoted to the analysis of old (classical) and new (non-Lie) symmetries of the fundamental equations of quantum mechanics and classical field theory, and to the classification and algebraic-theoretical deduction of equations of motion of arbitrary spin particles in both Poincaré invariant approach. The authors present detailed information about the representations of the Galilei and Poincaré groups and their possible generalizations, and expound a new approach for investigating symmetries of partial differential equations; this leads to finding previously unknown algebras and groups of invariance of the Dirac, Maxwell and other equations.

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Preface to the English Edition

"In the beginning was the symmetry"
W. Heisenberg

"Hidden harmony is stronger than the explicit one"
Heraclitus

The English version of our book is published on the initiative of Dr. Edward M. Michael, Vice-President of the Allerton Press Incorporated. It is with great pleasure that we thank him for his interest in our work.

The present edition of this book is an improved version of the Russian edition, and is greatly extended in some aspects. The main additions occur in Chapter 4, where the new results concerning complete sets of symmetry operators of arbitrary order for motion equations, symmetries in elasticity, super- and parasupersymmetry are presented. Moreover, Appendix II includes the explicit description of generalized Killing tensors of arbitrary rank and order: these play an important role in the study of higher order symmetries.

The main object of this book is symmetry. In contrast to Ovsiannikov's term "group analysis" (of differential equations) we use the term "symmetry analysis" in order to emphasize the fact that it is not, in general, possible to formulate arbitrary symmetry in the group theoretical language. We also use the term "non-Lie symmetry" when speaking about such symmetries which can not be found using the classical Lie algorithm.

In order to deduce equations of motion we use the "non-Lagrangian" approach based on representations of the Poincaré and Galilei algebras. That is, we use for this purpose the principles of Galilei and Poincaré-Einstein relativity formulated in algebraic terms. Sometimes we use the usual term "relativistic equations" when speaking about Poincaré-invariant equations in spite of the fact that Galilei-invariant subjects are "relativistic" also in the sense that they satisfy Galilei relativity principle.

Our book continues the series of monographs devoted to symmetries in mathematical physics. Moreover, we will edit "Journal of Nonlinear Mathematical Physics" which also will related to these problems.

We hope that our book will be useful for mathematicians and physicists in the English-speaking world, and that it will stimulate the development of new symmetry approaches in mathematical and theoretical physics.

Only finishing the contemplated work one understands how it was necessary to begin it
B. Pascal

Preface

Over a period of more than a hundred years, starting from Fedorov's works on symmetry of crystals, there has been a continuous and accelerating growth in the number of researchers using methods of discrete and continuous groups, algebras and superalgebras in different branches of modern natural sciences. These methods have a universal nature and can serve as a basis for a deep understanding of the relativity principles of Galilei and Poincaré-Einstein, of Mendeleev's periodic law, of principles of classification of elementary particles and biological structures, of conservation laws in classical and quantum mechanics etc.

The foundations of the theory of continuous groups were laid a century ago by Sofus Lie, who proposed effective algorithms to calculate
symmetry groups for linear and nonlinear partial differential equations. Today the classical Lie methods (completed by theory of representations of Lie groups and algebras) are widely used in theoretical and mathematical physics.

Our book is devoted to the analysis of old (classical) and new (non-Lie) symmetries of the basic equations of quantum mechanics and classical field theory, classification and algebraic theoretical deduction of equations of motion of arbitrary spin particles in both Poincaré and Galilei-invariant approaches. We present detailed information about representations of the Galilei and Poincaré groups and their possible generalizations, and expound a new approach to investigation of symmetries of partial differential equations, which enables to find unknown before algebras and groups of invariance of the Dirac, Maxwell and other equations. We give solutions of a number of problems of motion of arbitrary spin particles in an external electromagnetic field. Most of the results are published for the first time in a monographic literature.

The book is based mainly on the author's original works. The list of references does not have any pretensions to completeness and contains as a rule the papers immediately used by us.

We take this opportunity to express our deep gratitude to academicians N.N. Bogoliubov, Yu.A. Mitropol'ski, our teacher O.S. Parasiuk, correspondent member of Russian Academy of Sciences V.G. Kadyshevskii, professors A.A. Borgardt and M.K. Polivanov for essential and constant support of our researches in developing the algebraic-theoretical methods in theoretical and mathematical physics. We are indebted to doctors L.F. Barannik, I.A. Egorchenko, N.I. Serov, Z.I. Simenoh, V.V. Tretynyk, R.Z. Zhdanov and A.S. Zhukovski for their help in the preparation of the manuscript.

Introduction

The symmetry principle plays an increasingly important role in modern researches in mathematical and theoretical physics. This is connected with the fact that the basis physical laws, mathematical models and equations of motion possess explicit or unexplicit, geometric or non-geometric, local or non-local symmetries. All the basic equations of mathematical physics, i.e. the equations of Newton, Laplace, d'Alembert, Euler-Lagrange, Lame, Hamilton-Jacobi, Maxwell, Schrödinger etc., have a very high symmetry. It is a high symmetry which is a property distinguishing these equations from other ones considered by mathematicians.

To construct a mathematical approach making it possible to distinguish various symmetries is one of the main problems of mathematical physics. There is a problem which is in some sense inverse to the one mentioned above but is no less important. We say about the problem of describing of mathematical models (equations) which have the given symmetry. Two such problems are discussed in detail in this book.

We believe that the symmetry principle has to play the role of a selection rule distinguishing such mathematical models which have certain invariance properties. This principle is used (in the explicit or implicit form) in a construction of modern physical theories, but unfortunately is not much used in applied mathematics.

The requirement of invariance of an equation under a group enables us in some cases to select this equation from a wide set of other admissible ones. Thus, for example, there is the only system of Poincaré-invariant partial differential equations of first order for two real vectors E and H, and this is the system which reduces to Maxwell's equations. It is possible to "deduce" the Dirac, Schrödinger and other equations in an analogous way.

The main subject of the present book is the symmetry analysis of the basic equations of quantum physics and deduction of equations for particles of arbitrary spin, admitting different symmetry groups. Moreover we consider two-particle equations for any spin particles and exactly solvable problems of such particles interaction with an external field.

The local invariance groups of the basic equations of quantum mechanics (equations of Schrödinger, of Dirac etc.) are well known, but the proofs that these groups are maximal (in the sense of Lie) are present only in specific journals due to their complexity. Our opinion is that these proofs have to be expounded in form easier to understand for a wide circle of readers. These results are undoubtedly useful for a deeper understanding of mathematical nature of the symmetry of the equations mentioned. We consider local symmetries mainly in Chapter 1.

It is well known that the classical Lie symmetries do not exhaust the invariance properties of an equation, so we find it is necessary to expound the main results obtained in recent years in the study of non-Lie symmetries, super- and parasupersymmetries. Moreover we present new constants of motion of the basic equations of quantum physics, obtained by non-Lie methods. Of course it is interesting to demonstrate various applications of symmetry methods to solving concrete physical problems, so we present here a collection of examples of exactly solvable equations describing interacting particles of arbitrary spins.

The existence of the corresponding exact solutions is caused by the high symmetry of the models considered.

In accordance with the above, the main aims of the present book are:

1. To give a good description of symmetry properties of the basic equations of quantum mechanics. This description includes the classical Lie symmetry (we give simple proofs that the known invariance groups of the equations considered are maximally extensive) as well as the additional (non-Lie) symmetry.

2. To describe wide classes of equations having the same symmetry as the basic equations of quantum mechanics. In this way we find
the Poincaré-invariant equations which do not lead to known contradictions with causality violation by describing of higher spin particles in an external field, and the Galilei-invariant wave equations for particles of any spin which give a correct description of these particle interactions with the electromagnetic field. The last equations describe the spin-orbit coupling which is usually interpreted as a purely relativistic effect.

3. To represent hidden (non-Lie) symmetries (including super- and parasupersymmetries) of the main equations of quantum and classical physics and to demonstrate existence of new constants of motion which can not be found using the classical Lie method.

4. To demonstrate the effectiveness of the symmetry methods in solving the problems of interaction of arbitrary spin particles with an external field and in solving of nonlinear equations.

Besides that we expound in details the theory of irreducible representations of the Lie algebras of the main groups of motion of four-dimensional space-time (i.e. groups of Poincaré and Galilei) and of generalized Poincaré groups $P(1,n)$. We find different realizations of these representations in the bases available to physical applications. We consider representations of the discrete symmetry operators $P$, $C$ and $T$, and find nonequivalent realizations of them in the spaces of representations of the Poincaré group.

The detailed list of contents gives a rather complete information about subject of the book so we restrict ourselves by the preliminary notes given above.

The main part of the book is based on the original papers of the authors. Moreover we elucidate (as much as we are able) contributions of other investigators in the branch considered.

We hope our book can serve as a kind of group-theoretical introduction to quantum mechanics and will be interesting for mathematicians and physicists which use the group-theoretical approach and other symmetry methods in analysis and solution of partial differential equations.

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Additional list of references


Footnotes:

1 Fushchych = Fushchich (the first version is closer to the Ukrainian transcription)

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