APPLICATIONS OF CLASSICAL PHYSICS

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Preface

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This book is an introduction to the fundamentals and 21st-century applications of all the major branches of classical physics except classical mechanics, electromagnetic theory, and elementary thermodynamics (which we assume the reader has already learned elsewhere).

Classical physics and this book deal with physical phenomena on macroscopic scales: scales where the particulate natures of matter and radiation are secondary to the behavior of particles in bulk; scales where particles' statistical as opposed to individual properties are important, and where matter's inherent graininess can be smoothed over. In this book, we shall take a journey through spacetime and phase space, through statistical and continuum mechanics (including solids, fluids, and plasmas), and through optics and relativity, both special and general. In our journey, we shall seek to comprehend the fundamental laws of classical physics in their own terms, and also in relation to quantum physics. Using carefully chosen examples, we shall show how the classical laws are applied to important, contemporary, 21st-century problems and to everyday phenomena, and we shall uncover some deep connections among the various fundamental laws, and connections among the practical techniques that are used in different subfields of physics.

Many of the most important recent developments in physics—and more generally in science and engineering—involve classical subjects such as optics, fluids, plasmas, random processes, and curved spacetime. Unfortunately, many physicists today have little understanding of these subjects and their applications. Our goal, in writing this book, is to rectify that. More specifically:

- We believe that every masters-level or PhD physicist should be familiar with the basic concepts of all the major branches of classical physics, and should have had some experience in applying them to real-world phenomena; this book is designed to facilitate that.

- A large fraction of physics, astronomy and engineering graduate students in the United States and around the world use classical physics extensively in their research, and even more of them go on to careers in which classical physics is an essential component; this book is designed to facilitate that research and those careers.

- Many professional physicists and engineers discover, in mid-career, that they need an understanding of areas of classical physics that they had not previously mastered. This book is designed to help them fill in the gaps, and to see the relationship of topics they study to already familiar topics.

In pursuit of these goals, we seek, in this book, to *give the reader a clear understanding of the basic concepts and principles of classical physics.* We present these principles in the language of modern physics (not nineteenth century applied mathematics), and present them for physicists as distinct from mathematicians or engineers — though we hope that mathematicians and engineers will also find our presentation useful. As far as possible, we emphasize theory that involves general principles which extend well beyond the particular subjects we study.

In this book, we also seek to *teach the reader how to apply classical physics ideas.* We do so by presenting contemporary applications from a variety of fields, such as

- fundamental physics, experimental physics and applied physics,
- astrophysics and cosmology,
- geophysics, oceanography and meteorology,
- biophysics and chemical physics,
- engineering, optical science & technology, radio science & technology, and information science & technology.

Why is the range of applications so wide? Because we believe that physicists should have at their disposal enough understanding of general principles to attack problems that arise in unfamiliar environments. In the modern era, a large fraction of physics students will go on to careers away from the core of fundamental physics. For such students, a broad exposure to non-core applications will be of great value. For those who wind up in the core, such an exposure is of value culturally, and also because ideas from other fields often turn out to have impact back in the core of physics. Our examples will illustrate how basic concepts and problem solving techniques are freely interchanged between disciplines.

Classical physics is defined as the physics where Planck's constant can be approximated as zero. To a large extent, it is the body of physics for which the fundamental equations were established prior to the development of quantum mechanics in the 1920's. Does this imply that it should be studied in isolation from quantum mechanics? Our answer is, most emphatically, "No!". The reasons are simple:

First, quantum mechanics has primacy over classical physics: classical physics is an approximation, often excellent, sometimes poor, to quantum mechanics. *Second*, in recent decades many concepts and mathematical techniques developed for quantum mechanics have been imported into classical physics and used to enlarge our classical understanding and enhance our computational capability. An example that we shall discuss occurs in plasma physics, where nonlinearly interacting waves are treated as quanta ("plasmons"), despite the fact that they are solutions of classical field equations. *Third*, ideas developed initially for "classical" problems are frequently adapted for application to avowedly quantum mechanical subjects; examples (not discussed in this book) are found in supersymmetric string theory and in the liquid drop model of the atomic nucleus. Because of these intimate connections between quantum and classical physics, quantum physics will appear frequently in this book, in many ways.

The amount and variety of material covered in this book may seem overwhelming. If so, please keep in mind the key goals of the book: to teach the fundamental concepts, which are not so extensive that they should overwhelm, and to illustrate those concepts. Our goal is *not* to provide a *mastery* of the many illustrative applications contained in the book, but rather to convey the spirit of how to apply the basic concepts of classical physics. To help students and readers who feel overwhelmed, we have labeled as “Track Two” sections that can easily be skipped on a first reading, or skipped entirely — but are sufficiently interesting that many readers may choose to browse or study them. Track-Two sections are labeled by the symbol **T2**. To keep Track One manageable for a one-year course, the Track-One portion of each chapter is no longer than 40 pages (including many pages of exercises) and often somewhat shorter.

This book will also seem much more manageable and less overwhelming when one realizes that the same concepts and problem solving techniques appear over and over again, in a variety of different subjects and applications. *These unifying concepts and techniques are listed in outline form in Appendix B, along with the specific applications and section numbers in this book, where they arise.* The reader may also find Appendix A useful. It contains an outline of the entire book based on concepts — an outline complementary to the Table of Contents.

This book is divided into seven parts; see the Table of Contents:

I. Foundations — which introduces a powerful *geometric* point of view on the laws of physics (a viewpoint that we shall use throughout this book), and brings readers up to speed on some concepts and mathematical tools that we shall need. Many readers will already have mastered most or all of the material in Part I, and may find that they can understand most of the rest of the book without adopting our avowedly geometric viewpoint. Nevertheless, we encourage such readers to browse Part I, at least briefly, before moving onward, so as to become familiar with our viewpoint. It does have great power.

Part I is split into two chapters: Chap. 1 on Newtonian Physics; Chap. 2 on Special Relativity. Since the vast majority of Parts II–VI is Newtonian, readers may choose to skip Chap. 2 and the occasional special relativity sections of subsequent chapters, until they are ready to launch into Part VII, General Relativity. Accordingly Chap. 2 is labeled Track Two, though it becomes Track One when readers embark on Part VII.

II. Statistical physics — including kinetic theory, statistical mechanics, statistical thermodynamcs, and the theory of random processes. These subjects underly some portions of the rest of the book, especially plasma physics and fluid mechanics. Among the applications we study are the statistical-theory computation of macroscopic properties of matter (equations of state, thermal and electric conductivity, viscosity, ...); phase transitions (boiling and condensation, melting and freezing, ...); the Ising model and renormalization group; chemical and nuclear reactions, e.g. in nuclear reactors; Bose-Einstein condensates; Olber’s Paradox in cosmology; the Greenhouse effect and its influence on the earth's climate; noise and signal processing, the relationship between information and entropy; entropy in the expanding universe; and the entropy of black holes.

III. Optics — by which we mean classical waves of all sorts: light waves, radio waves, sound waves, water waves, waves in plasmas, and gravitational waves. The major concepts we develop for dealing with all these waves include geometric optics, diffraction, interference, and nonlinear wave-wave mixing. Some of the applications we will meet are gravitational lenses, caustics and catastrophes, Berry's phase, phase-contrast microscopy, Fourier-transform spectroscopy, radio-telescope interferometry, gravitational-wave interferometers, holography, frequency doubling and phase conjugation in nonlinear crystals, squeezed light, and how information is encoded on BD's, DVD's and CD's.

IV. Elasticity — elastic deformations, both static and dynamic, of solids. Here some of our applications are bifurcations of equilibria and bifurcation-triggered instabilities, stress-polishing of mirrors, mountain folding, buckling, seismology and seismic tomography, and elasticity of DNA molecules.

V. Fluid Dynamics — with the fluids including, for example, air, water, blood, and interplanetary and interstellar gas. Among the fluid concepts we study are vorticity, turbulence, boundary layers, subsonic and supersonic flows, convection, sound waves, shock waves and magnetohydrodynamics. Among our applications are the flow of blood through constricted vessels, the dynamics of a high-speed spinning baseball, how living things propel themselves, convection in stars, helioseismology, supernovae, nuclear explosions, sedimentation and nuclear winter, the excitation of ocean waves by wind, salt fingers in the ocean, tornados and water spouts, the Sargasso Sea and the Gulf Stream in the Atlantic Ocean, nonlinear waves in fluids (solitons and their interactions), stellerators, tokamaks, and controlled thermonuclear fusion.

VI. Plasma Physics — with the plasmas including those in earth-bound laboratories and technological devices, the earth's ionosphere, stellar interiors and coronae, and interplanetary and interstellar space. In addition to magnetohydrodynamics (treated in Part V), we develop three other physical and mathematical descriptions of plasmas: kinetic theory, two-fluid formalism, and quasi-linear theory which we express in the quantum language of weakly coupled plasmons and particles. Among our plasma applications are: some of the many types of waves (plasmons) that a plasma can support—both linear waves and nonlinear (soliton) waves; the influence of the earth's ionosphere on radio-wave propagation; the wide range of plasma instabilities that have plagued the development of controlled thermonuclear fusion; and wave-particle (plasmon-electron and plasmon-ion) interactions, including the two-stream instability for fast coronal electrons in the solar wind, isotropization of cosmic rays via scattering by magnetosonic waves, and Landau damping of electrostatic waves.

VII. General Relativity — the physics of curved spacetime, including the laws by which mass-energy and momentum curve spacetime, and by which that curvature influences the motion of matter and infliuences the classical laws of physics (e.g., the laws of fluid mechanics, electromagntic fields, and optics). Here our applications include, among others, gravitational experiments on earth and in our solar system; relativistic stars and black holes, both spinning (Kerr) and nonspinning (Schwarzschild); the extraction of spin energy from black holes; interactions of black holes with surrounding and infalling matter; gravitational waves and their generation and detection; and the large-scale structure and evolution of the universe (cosmology), including the big bang, the inflationary era, and the modern era. Throughout, we emphasize the physical content of general relativity and the connection of the theory to experiment and observation.

This book's seven Parts are semi-independent of each other. It should be possible to read and teach the parts independently, if one is willing to dip into earlier parts occasionally, as needed, to pick up an occasional concept, tool or result. We have tried to provide enough cross references to make this possible.

Track One of the book has been designed for a full-year course at the first-year graduate level; and that is how we have used it, covering Part I in the first week, and then on average one chapter per week thereafter. (Many fourth-year undergraduates have taken our course successfully, but not easily.)

Exercises are a major component of this book. There are five types of exercises:

1. *Practice.* Exercises that give practice at mathematical manipulations (e.g., of tensors).

2. *Derivation.* Exercises that fill in details of arguments or derivations which are skipped over in the text.

3. *Example.* Exercises that lead the reader step by step through the details of some important extension or application of the material in the text.

4. *Problem.* Exercises with few if any hints, in which the task of figuring out how to set the calculation up and get started on it often is as difficult as doing the calculation itself.

5. *Challenge.* An especially difficult exercise whose solution may require that one read other books or articles as a foundation for getting started.

We urge readers to try working many of the exercises, and *read and think about all of the Example exercises.* The Examples should be regarded as continuations of the text; they contain many of the most illuminating applications. We label with double stars, **, Example exercises that are especially important.

A few words on **units:** In this text we will be dealing with practical matters and will frequently need to have a quantitative understanding of the magnitudes of various physical quantities. This requires us to adopt a particular unit system. Students we teach are about equally divided in preferring cgs/Gaussian units or MKS/SI units. Both of these systems provide a complete and internally consistent set for all of physics and it is an often-debated issue as to which is the more convenient or aesthetically appealing. We will not enter this debate! One's choice of units should not matter and a mature physicist should be able to change from one system to another with only a little thought. However, when learning new concepts, having to figure out "where the 4π's go" is a genuine impediment to progress. Our solution to this problem is as follows: We shall use the units that seem most natural for the topic at hand or those which, we judge, constitute the majority usage for the subculture that the topic represents. We shall not pedantically convert cm to m or *vice versa* at every juncture; we trust that the reader can easily make whatever translation is necessary. However, where the equations are actually different, for example in electromagnetic theory, we shall sometimes provide, in brackets or footnotes, the equivalent equations in the other unit system and enough information for the reader to proceed in his or her preferred scheme. As an aid, we also give some unit-conversion information in Appendix C, and values of physical constants in Appendix D.

We have written this book in connection with a full-year course that we and others have taught at Caltech nearly every year since the early 1980s. We conceived that course and this book in response to a general concern at Caltech that our PhD physics students were being trained too narrowly, without exposure to the basic concepts of classical physics beyond electricity and magnetism, classical mechanics, and elementary thermodynamics. Courses based on parts of this book, in its preliminary form, have been taught by various physicists, not only at Caltech but also at a few other institutions in recent years, and since moving to Stanford in 2003, Blandford has taught from it there. Many students who took our Caltech course, based on early versions of our book, have told us with enthusiasm how valuable it was in their later careers. Some were even enthusiastic during the course.

Many generations of students and many colleagues have helped us hone the book's presentation and its exercises through comments and criticisms, sometimes caustic, usually helpful; we thank them. Most especially:

For helpful advice about presentations and/or exercises in the book, and/or material that went into the book, we thank Professors Richard Blade, Yanbei Chen, Michael Cross, Steven Frautschi, Peter Goldreich, Steve Koonin, Sterl Phinney, David Politzer, and David Stevenson at Caltech (all of whom taught portions of our Caltech course at one time or another), and XXXXX [ROGER: WHO ELSE SHOULD WE BE LISTING?]

Over the years, we have received extremely valuable advice about this book from the teaching assistants in our course: XXXXXXX[KIP IS ASSEMBLING A LIST]XXXXXXXX We are very indebted to them.

We hope that this book will trigger a significant broadening of the training of physics graduate students elsewhere in the world, as it has done at Caltech, and will be of wide use to mature physicists as well.

Roger D. Blandford and Kip S. Thorne
Stanford University and Caltech, December 2012

CONTENTS

[For an alternative overview of this book, See *Appendix A. Concept-Based Outline* (does not exist yet)]

3.5 Isotropic Distribution Functions and Equations of State: density, pressure, energy density, equation of state for nonrelativistic hydrogen gas, for relativistic degenerate hydrogren gas, and for radiation

3.6 Evolution of the Distribution Function: Liouville's Theorem, the Collisionless Boltzmann Equation, and the Boltzmann Transport Equation

Box: [T2] Sophisticated derivation of relativistic collisionless Boltzmann equation

**Examples: Solar heating of the earth—the Greenhouse effect; Olber's paradox and solar furnace

3.7 Transport Coefficients: diffusive heat conduction inside a star, analyzed in order of magnitude and via the Boltzmann transport equation

Box: Two lengthscale expansions

**Examples: Solution of diffusion equation in an infinte, homogeneous medium; Diffusion equation for temperature; Viscosity of a monatomic gas; Neutron diffusion in a nuclear reactor

4. Statistical Mechanics

4.1 Overview

4.2 Systems, Ensembles, and Distribution Functions

Box: [T2] Density operator and quantum statistical mechanics

4.3 Liouville's Theorem and the Evolution of the Distribution Function

4.4 Statistical Equilibrium: canonical ensemble and distribution; general ensemble; Gibbs ensemble; grand canonical ensemble; Bose-Einstein and Fermi-Dirac distributions; equipartition theorem

4.5 The Microcanonical Ensemble

4.6 The Ergodic Hypothesis

4.7 Entropy and the Evolution into Statistical Equilibrium: the second law of thermodynamics; what causes entropy to increase?

Box: [T2] Entropy increase due to discarding quantum correlations

**Exercises: Entropy of a thermalized mode of a field; Entropy of mixing, indistinguishability of atoms, and the Gibbs paradox

4.8 Entropy Per Particle

**Exercise: Primordial element formation

4.9 Bose-Einstein Condensate

**Exercise: Onset of Bose-Einstein condensation; Discontinuous change of specific heat

III. OPTICS

7. Geometrical Optics

7.4 Paraxial Optics

Applications: lenses and mirrors, telescope, microscope, optical cavity, converging magnetic lens for charged particle beam

7.5 Catastrophe Optics: Multiple Images; Formation of Caustics and their Properties

Applications: Imperfect lens, sunlight's caustic patterns on the bottom of a swimming pool and through a water glass

**Exercises: Catastrophe theory – the five elementary catastrophes applied to optical caustics and to the van der Waals phase change

7.6 [T2] Gravitational Lenses: Refractive index model; Multiple Images and Caustics

[T2] Applications: Lensing by a point mass; lensing of a quasar by a galaxy

7.7 Polarization: its geometric-optics propagation

[T2] Application: The Geometric Phase

8. Diffraction

8.1 Overview

8.2 Helmholtz-Kirchhoff Integral: diffraction by an aperture; spreading of the wavefront

8.3 Fraunhofer Diffraction: diffraction grating; Airy pattern; Hubble space telescope; Babinet's principle

Other Applications: Pointilist paintings; light scattering by large opaque particle; measuring thickness of human hair via diffraction

8.4 Fresnel Diffraction: Fresnel integrals and Cornu spiral; lunar occultation of a radio source; circular apertures, Fresnel zones and zone plates

Other Applications: seeing (stellar scintillation) in the atmosphere; multiconjugate adaptive optics; spy satellites

8.5 Paraxial Fourier Optics: coherent illumination; point spread functions; Abbé theory of image formation by a thin lens; spatial filtering in the focal plane of a lens; Gaussian beams; gravitational-wave interferometers

Other applications of paraxial Fourier optics — high-pass filter to clean a laser beam; low-pass filter to enhance sharp features in an image; notch filter to remove pixellations from an image; phase contrast microscopy; spatial pattern recognition; convolution; Gaussian beams in an optical fiber; scattered-light noise in LIGO

**Example: Transmission electron microscope

8.6 Diffraction at Caustics; scaling laws

APPENDICES

Part 2. Thermodynamics, Statistical Physics, and Quantum Mechanics -S.Cahn, B.Nadgorny Applications of Classical Physics - Blandford R.D., Thorne K.S. CRC Press - Dictionary of Pure and Applied Physics - D. Basu Essential Physics 1 - F. Firk How To Solve Physics Problems - R. Oman, D. Oman Motion Mountain - The Adventure of Physics - C. Schiller Physical Science Concepts - Mason, Griffen, Merrill, Thorne Physics 2000.Â (College Text) - Serway and Jewett Physics Formulary - J. Wevers Principles of Modern Physics - N. Ashby, S. Miller Schaum's Easy Outlines - College Physics Crash Course - F. Bueche, E. Hecht Schaum's Outline of Applied Physics - Arthur Beiser Basically, classical physics refers to fields of physics before quantum mechanics and Einsteinâ€™s relativityâ€”both of which belong to modern physicsâ€”A lot of classical physics are Newton, as he is a giant in theoretical physics (I should remind the reader that in Newtonâ€™s time, most common people still believed that the Earth was the center of the solar system; gravity.Â The key breakthrough that established Classical Physics was the application of mathematics to build models which could be used to represent these macroscopic phenomena. This breakthrough is associated with Newton but built upon the works of others including Copernicus and Galileo.