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The Variability of Student Reasoning, Lecture 3: Manifold Cognitive Resources

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The previous lectures focused on phenomenology: What sorts of occurrences do we see in students' reasoning? This third and final lecture focuses on ontology: What sorts of things do we attribute to students' minds? It has become conventional to speak and think in terms of conceptions, naïve theories, and stages of development. These are all attributions of stable properties, and they account well for patterns that can occur in student reasoning. They do not account well, however, for the variability and multiple patterns illustrated in the previous lectures. Research in cognitive science provides an alternative ontology of multiple, fine-grained cognitive resources that are context-sensitive in their activation. This lecture reviews some of that work and draws implications for elementary science education.

1 Introduction

The previous lectures focused on phenomenology: What sorts of occurrences do we see in students' reasoning? The first reviewed examples of children's inquiries, highlighting productive ways of thinking. The second reviewed examples of university students' not so productive ways of thinking. It also illustrated how children and older students can be observed to reason in multiple ways, reviewing examples of transitions in substance and approach over short time scales. Some transitions were prompted by particular suggestions for how to think of knowledge and learning; others occurred without any specific intervention.

The many examples show the richness and complexity of possibilities in students' knowledge and reasoning. When teachers establish classroom environments that focus on children's thinking, children are often capable of a range of ideas and approaches that includes much of value in science. Within university physics courses, instructors often see a narrower range in their students that excludes everyday logic and common sense.

This lecture turns to ontology: What sorts of things do we attribute to students' minds?[1] The practice among science educators has been primarily to attribute knowledge in the form of *conceptions* and to think of reasoning abilities as determined by stages of development. I refer to these views as "unitary," because they attribute units of cognition to particular areas of thought. With respect to dynamics, for example, educators model student reasoning as governed by the conception that *forces cause motion*. In the following section, which makes up most of the lecture, I review how research in cognitive science has been moving from unitary models to models of minds as made up of many smaller parts: manifold ("many-fold") views of cognitive structure. A manifold view of student reasoning about dynamics sees knowledge about dynamics as comprised of many smaller pieces and not generally governed by any one. In the final section I argue that this shift from unitary to manifold views has significant implications for objectives and assessment in elementary science education.

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[1] The word "ontology" means "knowledge about the nature of being." Here the term refers to understandings of what sorts of entities "exist" in minds.

2 From unitary to manifold

This section opens with an argument that the perspective of misconceptions prevalent in science education reflects a confusion of phenomenology and ontology. It then turns in part 2.2 to a brief review of recent research that attends to variability in student knowledge and reasoning, arguing for manifold views of cognitive structure. Part 2.3 focuses on epistemologies. 2.4 discusses how this view is evident in expert instructional practices, and 2.5 outlines questions for further research.

2.1 The perspective of misconceptions: A confusion of phenomenology and ontology

I have taken care to distinguish phenomenology (What sorts of occurrences do we see in students' reasoning?) from ontology (What sorts of things to we attribute to their minds?) because educators often seem to confuse them. In many circumstances students tend to associate motion with force: Objects move if they are pushed; they are stationary if they are not; a constant force produces or is equivalent to constant velocity. Recognizing this and other patterns has been important progress, including for what we attribute to students' minds: Evidence of tendencies in naïve reasoning about dynamics [1-3] motivates attributing knowledge of dynamics in some form. *Something* must produce these patterns.

From there, though, the recognition of phenomena has migrated in usage and thought to attribution of cognitive structure, evident and perhaps partly born out of colloquial references to students "having" or "holding on to" misconceptions. In this case, a recurrent tendency for associating motion and force has become reified as an entity of mind, the misconception as a cognitive structure, in some descriptions afforded the status of intuitive "theory" or "framework." This step to ontology produces a view of naïve physics as rife with obstacles—aberrant cognitive objects—to the development of expertise.

Many who promote a misconceptions perspective argue that they do not intend it as a purely deficit account. To the contrary, they intend it to show the rationality of students' reasoning, and some choose the terms "preconceptions" or "alternative conceptions" to avoid the derogatory prefix "mis."[2] The heart of the issue, however, is not the evaluative prefix but the idea of a conception as a unit of cognitive structure: If students "have" conceptions that conflict with expertise, then those conceptions must be changed or removed.

That seems to be what the broader community understands. At the time of this writing, a Google search on "physics misconceptions" produces (along with sites focused on textbook inaccuracies) sites devoted to helping instructors identify and "remedy" misconceptions. The authors of the "Physics Misconceptions Center,"[3] the first of these on the list, "have read through Physics Education Research literature and compiled a list of documented student misconceptions and difficulties, along with questions that you can use in your classroom to discover if your students have these misconceptions and/or difficulties." The next link is a pdf document titled "Eradicating physics misconceptions using the Conceptual Change Method."[4] The next[5] lists "clarifications of certain physics misconceptions or preconceptions," with videoclips "to prove these misconceptions (or preconceptions) are incorrect."

Research to identify misconceptions continues apace. It motivates attention to students' prior knowledge in instruction, but that attention is corrective. The ever-growing catalog of misconceptions is a list of defects to repair. Meanwhile, the challenge of repairing those defects raises theoretical questions.

Most thinking on instructional methods can be traced to educators' interpretations of Piagetian theory, ideas of cognitive dissonance and accommodation. For conceptual change to occur, students must become dissatisfied with their current conceptions — experience some sort of conflict around them — and then see a way out of that conflict in other, new conceptions [5]. That last step,

[2] Clement and his colleagues have specifically discussed productive preconceptions, that can contribute to expert understanding [4],and their account of "bridging analogies" moves toward a more nuanced view of students' prior knowledge.

[3] www.physics.montana.edu/physed/misconceptions/

[4] www.kzoo.edu/educ/PodolnerSIP.pdf

[5] schools.moe.edu.sg/xinmin/lessons/physics/misconceptions/physicsmisconceptions.htm

researchers have observed, presents a paradox: If students process new information through their existing conceptual structures, how can they see the way to new conceptions [6]? This is a problem specifically for the misconceptions perspective, that it offers no insight into what productive knowledge students use to construct more expert understanding [7]. It is also a problem for stage-based theories of cognitive development (see below) that there is no clear mechanism for how new stages form [8].

Phenomenological patterns need not align with cognitive structure, however, and evidence of variability [3,8,9] contradicts attributions of conceptions as ontological units. Students do often think of motion as caused by force, but it is easy to pose questions about familiar situations in which they do not. It is useful to consider misconceptions as phenomenological patterns, but evidence of variability argues against understanding them as fixed structural attributes. What then should we think of, when we think of students' knowledge and reasoning?

2.2 Manifold resources for knowledge and reasoning

2.2.1 Intuitive knowledge

An analogy to nuclear physics may be useful to frame this discussion. For about half of the time that we've known about protons we have thought of them as fundamental units. During the 1960's, the phenomenology of deep inelastic scattering gave reason to question that view, and physicists began to think in terms of substructure to the proton (and neutron and other particles involved in strong interactions), positing smaller constituent particles. Not having a sense yet of the specific nature of those underlying particles, but wanting to pursue the implications of thinking they exist, they used the deliberately generic term "parton." The term "resources" is similarly generic here, reflecting a modest theoretical step to think of smaller parts than conceptions or naïve theories. Like the parton model it remains non-committal regarding specific properties of the parts.

A resources view of knowledge and reasoning differs in three key respects from a view of conceptions. First, resources are elements of cognitive function rather than declarative statements about the world. For example, a conception may be "force causes motion," a statement that physicists consider false. In contrast, the resource "maintaining agency" is an element of sense of mechanism involving an agent (not necessarily force) continuously maintaining an effect (not necessarily motion). The resource itself cannot be assessed as true or false; like a tool, it is useful in some situations and not in others.

Second, resources are context-sensitive in their activation, rather than applied generally. To continue with the same example, the conception *force causes motion* is understood as part of how the mind interprets experience and information about forces and motion. There is not an account of "activating" or "deactivating" this knowledge; it is a stable feature, expected to filter and organize all perceptions and reasoning that pertains to motion. In contrast, *maintaining agency* would be activated in some situations, such as in understanding how an oven stays hot or an automobile keeps moving, but not in others, such as in understanding why a yellow shirt remains yellow or why steam rises.[6]

Finally, resources are manifold and diverse. Rather than attribute a single unit of mind to a particular area of thought, this view attributes a variety of resources that may apply. Here I have been using the relationship between force and motion as an example of the area. Rather than attribute one conception we would posit a variety of resources for thinking about the relationship, including *maintaining agency, actuating agency*, *Ohm's p-prim, force as spinner, dying away*, all of which are "p-prims" (phenomenological primitives) from diSessa's account [10,11].

I have been drawing heavily on diSessa's account, which he presents in this volume [11]. *Sucking* is another of his core examples; on this view some of Ms. Kagey's students in lecture 1 were activating *sucking* in their thinking about how red dye might make its way up a carnation stem. This p-prim needs an agent, and so the students were trying to figure out what that agent might be. Other students were activating thinking in terms of *absorbing*, a primitive we could posit that doesn't need

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[6] That is, one might think of yellow as a property of the shirt, not needing any maintaining cause. And one might think of it as a property of steam to rise, again without needing any maintaining cause. Without *maintaining agency* active, there is not a sense of any cause one could remove to end the effect.

an agent: A sponge absorbs by virtue of being a sponge, but someone or something needs to suck on a straw.

Maintaining agency and *Actuating agency* (an agent initiates an effect, but the effect outlasts the cause) would be implicated in the 5th/6th graders' thinking about a pendulum. *Resistance* would be implicated in students' thinking about the effect of air on the motion of paper or keys, *Blocking* in one of Ms. Mikeska's students' ideas about how sand or soil might keep water from getting to a seed.

diSessa's "coordination classes" are larger, models of the cognitive structure of particular concepts such as *force*, *event*, or *object* [11,12]; see also Mestre's lecture [13], an application of that construct to model a particular variation in student reasoning about motion. P-prims and coordination classes model specific types of conceptual resources, but there would need to be many other types as well. The concept of conservation, evident in the 3rd grade discussion of ice or water melting and in the 8th graders reasoning about beads or salt in water, would not be well described by either p-prims or coordination classes. Of course, there have been a variety of other accounts of manifold cognitive structures [14-16].

2.2.2 Reasoning abilities

Traditional theories of cognitive development depict a progression of reasoning abilities in stable, equilibrated stages. These have motivated taxonomies of scientific "process skills," progressing from more "concrete" abilities, such as of "observing," "classifying," and "measuring," to more "abstract" abilities, such as of "formulating hypotheses," "designing experiments," and "critiquing theories." By these views, children should only be engaged in activities that are developmentally appropriate, concrete tasks for younger children, more abstract tasks as they get older. As well, because developmental levels are expected to be general across contexts and domains, curricula can be designed to target particular abilities.

Research in cognitive psychology has been eroding at ideas of universal stages of development for some time, with evidence and theoretical arguments in favor of more variegated accounts of children's reasoning [8,17-19]. There is a growing consensus that standard depictions of equilibrated stages misrepresent and generally underestimate children's abilities, and that the skills in these taxonomies cannot be seen as unitary constructs: Abilities to classify or formulate hypotheses vary with context.

We have our own evidence of problems with these developmental accounts in the examples from lecture 1, in which young children showed signs of what stage-based theories would not expect until later years. As early as 1st grade we could see children keeping track of different perspectives (Eleanor's noting that Tammy's idea was the same as Maxwell's; 1st graders restating each other's ideas in discussing falling objects). Second graders noticed and tried to reconcile inconsistencies in their sense of mechanism (how could magnetic power flow through a hand without it hurting?) and between evidence and theory (metal in an x-ray is probably what the person is wearing, not metal inside the bones). Third graders were formulating, working with, and critiquing the merits of analogies and analogical models (the ice-in-water analogy for a rock in lava; the analogical model of sponges for how fluid moves up a carnation stem). These examples add to many other studies of student inquiry within classroom contexts that show greater variability in children's [20] or older students' [21]abilities than stage-based theories describe.

Still, the lists of process skills organized by developmental level remain the main basis for thinking about objectives and assessments with respect to science as inquiry. They are not without value as an initial take on aspects of children's thinking we should consider relevant, but they greatly underestimate the richness and diversity of possibilities. At present, however, there is little to supplant these lists as a framework. We need better conceptualizations for learning science as inquiry.

One aspect of that need is in the conceptualization of what we consider as science. Kuhn's work [22] has been valuable progress in this regard, identifying the importance of argumentation as central to scientific inquiry, specifically the coordination of theory and evidence. Another direction for further progress may be in the study of scientists' reasoning, current and historical. There has been a great deal of thought with respect to comparing students' ideas to historical theories [2,23]; there has been less with respect to comparing students' and scientists' approaches to reasoning [24].

A second need is for theoretical development regarding the nature of these abilities and how they develop. Kuhn's [22] work identifies an area for attention, but it suffers from the same concerns as other unitary models, that it does not account for the richness and variability of the phenomena: Children's abilities to coordinate multiple theoretical perspectives depend on context. Samarapungavan [25], for example, found more sophisticated abilities than Kuhn's framework describes when she asked 7 and 11 year-old children to evaluate alternative theories about moon phases. Asked whether data agree or disagree with what "Ann" or "Joe" thinks, children show abilities they may not if asked the same question without reference to individuals.

Karmiloff-Smith [18] has discussed development in terms of "reflective redescription" that begins from an implicit level at which an individual shows evidence of having some ability but no awareness of it. In one experiment, children are shown two situations, one of a boy doll with one car, one pencil, and three balls, the other of a girl doll with three cars, one book, and one ball. Presented the statements "Lend me the car" or "Lend me a car," 3 and 4 year-old children had no difficulty inferring which doll the speaker would be addressing, evidence they have resources for distinguishing definite and indefinite articles. But they showed no awareness of how they made the inference: Asked how they knew which doll it was, children would give irrelevant answers, such as that the girl likes cars. Siegler [8] has similarly provided evidence of children's employing reasoning strategies, such as for arithmetic computation [26], without conscious access to what they are doing. These studies focus on early development, but research to identify resources for scientific reasoning in older students and scientists may also expect an implicit level, perhaps evident in studies of non-verbal reasoning, such as of "imagistic simulations" [27] or gestures [28].

In Karmiloff-Smith's account [18], children eventually become aware of these implicit resources. At the first level of explicit awareness (E1), in her framework, the knowledge remains inarticulate: Subjects are aware of how they are thinking, but they cannot yet articulate it. Further redescription leads to higher levels of explicitness (E2, E3), at which subjects become articulate about their knowledge or abilities. This progression is always occurring, not universally as stages of development but in many more localized developments. The E levels in Karmiloff-Smith's framework correspond to what we have begun to describe as "epistemological resources."

2.3 Epistemological resources

Research on student epistemologies has followed a similar course to research on conceptual knowledge and reasoning abilities. Evidence of tendencies in students' approaches to learning and comments about knowledge and reasoning [29-32] motivates attributing epistemology[7] to students in some form: Something must produce these patterns. Again, however, research has proceeded largely on the assumption that ontological structure matches those patterns. Perry's[29] scheme, the basis of most subsequent research , followed developmental psychology in modeling epistemologies as progressing in stages of development. More recently the field has shifted to emulating research on conceptual change in attributing "beliefs" or "theories" as units of cognitive structure [32].

As before, evidence of multiplicity in personal epistemologies contradicts unitary accounts of cognitive structure [33,34]. Students in some circumstances may tend to think of knowledge as coming from authority, but in other circumstances they may think of it as invented or inferred. Our work on epistemological resources [35,36]started with the observation that a five-year-old child is able to answer the question "how do you know?" in a variety different ways. Depending on the particular situation, the child could answer that she knows because someone told her, she saw it herself, she figured it out, she made it up, she guessed, and so on. Moreover, there is no reason to believe that children systematically think of knowledge in one way rather than another; in fact, we take it for granted that children can navigate across contexts with relative ease. Were a five-year-old systematically to attribute his knowledge to authority, such as if asked "how do you know you have ten fingers," we would notice and take it as cause for concern.

[7] For philosophers the term "epistemology" should be reserved for formal, articulate theories about knowledge; research in education and psychology has appropriated the term to refer to informal, possibly inarticulate knowledge about knowing.

There can be patterns within particular contexts, however. Thinking of a child, it is easy to imagine conversations in which "how do you know" would tend to have one answer or another. For example, in a conversation organized around story-telling, a child may be likely to answer along the lines of "I made it up," or "I remember"; in a conversation organized around story-comprehension, the child may tend to refer to what the narrator said. Similarly for students, we find that different contexts engender different patterns. For this reason, it is essential to attend to context in characterizing "beliefs" in individuals, and it is problematic to give them ontological status as units of cognitive structure [31].

Modeling the form epistemologies take in the mind, it is important to account for both the observed patterns within contexts and for the variations and flexibility. Manifold accounts in the literature include Lakoff's and Johnson's [37] analysis of metaphors for understanding knowledge, for example the common metaphor of "words as containers" of meaning. Collins and Ferguson [38] analyzed "epistemic forms" and "epistemic games" that comprise or contribute to expertise. Minksy's [14] computational model of mind as "society" includes a variety of epistemological "agents." Drawing on this work, we set out to develop an account of epistemological resources [35,36].

2.3.1 Resources for understanding sources of knowledge

We began to posit particular resources by thinking of a child's choices for responding to "how do you know," which suggests a range of possibilities.

Propagated stuff: Activating this resource means thinking of knowledge as a kind of stuff passed from a source to a recipient. Knowledge is not, in this sense, like money or material; it is more like a state (of sickness, for example).

Fabricated stuff: Knowledge is constructed or inferred from some source material. Asked "How do you know that 3 x 7 = 21?" a child may answer "because 3 x 15 is 15, and 7 is 5 + 2," describing the source material for the idea. With this resource, knowledge is made from other stuff; one characteristic is that anyone with access to that same stuff could do the same for themselves. (In that example, it would be easy to imagine the child applying *Propagate stuff* instead to say "my teacher told me.")

Free creation: Activating this resource means thinking of knowledge as invented freely. Children often understand their own imaginations as the source; "I made it up" is the obvious answer to "how do you know" in familiar situations.

Direct perception: "I know you have my toy because I can see it in your hand!"

Stored and retrieved: This attributes knowledge to the mind, as the proximal source—"I remember" or "I know from experience."

An individual might activate one or more of these resources in understanding the source of knowledge in any particular moment. They may also be refined with use, in patterns of activation that become resources in their own right. Physicists come to understand knowledge as *derived* for example, or as *empirical*, ideas we could unpack in terms of earlier elements—knowledge as *derived* a refinement (with other resources) of *fabricated stuff*.

2.3.2 Forms, activities, states

Thinking of various resources for understanding sources of knowledge, we do not suppose we have a complete list, or even that all of the resources we have suggested will withstand scrutiny. It does seem manageable to produce a list, however, and with respect to sources of knowledge we can at least venture a set.

There must be many other resources for understanding knowledge and knowing, including forms of knowledge, knowledge related activities, and states of knowing. Producing a list quickly begins to seem an unmanageable task! Inuit languages, the story goes, have many more words for snow than European languages because Inuit peoples have much more experience with snow in its many forms. Now consider the vocabulary in our languages for describing and referring to knowledge. Regarding forms of knowledge, we have *lists, pictures, graphs, stories, facts, rules, questions, statements, categories, puzzles, problems, names, numbers,* and so on, all of which may not belong in the same category. We have also *purposes, excuses, reasons, assertions, orders, objections, theories, hypotheses, evidence,* etc.

The same applies to resources for understanding knowledge-related activities. Again, we have a very rich vocabulary, including *brainstorming, guessing, checking, composing, correcting, imagining, trying to remember, formulating,* etc. We can also speak of various states of mind, including *doubt, understanding, puzzlement, belief, disbelief, acceptance.* Thinking again of Karmiloff-Smith's [18] model of progress from implicit abilities to articulate redescriptions, this richness of explicit vocabulary is more reason to expect richness in an account of reasoning abilities.

Working out the specifics of a resources model with respect to knowledge-related forms and activities will be quite difficult. Much of the work overlaps with linguistics, and we fall quickly into the theoretical challenges of understanding human categorization [39], where there has been much controversy, even with respect to the phenomenology and ontology of categories of color, which at least at first glance ought to be a simpler domain than epistemology. Those problems aside, it quickly becomes apparent that generating a complete list of epistemological resources is a daunting task, and too long a list would not be useful in practice. As with diSessa's p-prims, it may have to suffice for now to describe a small number of these resources, and posit others as needed.

For now, thinking in terms of epistemological resources may mean supposing resources that roughly align with vocabulary. Of course, that is a risky thing to do: The same move applied to *force* would get us into trouble, since that word in different uses reflects activation of different meanings. And it is already clear that the word *story* can have different meanings in different uses; it would not make sense to posit a strict correspondence. Still, it will allow us to proceed with exploring general implications of the resources ontology, and that may be productive as long as we are careful not to draw inferences that hinge on the specifics.

2.3.3 Considerations for instruction

Watching and listening to students, we can think about their understandings of knowledge and reasoning.

In the 1st grade example in lecture 1 about whether a seed can grow in sand, Eleanor commented that Tammy's idea that "the seed won't grow because there's not food" was "similar" to Maxwell's: "He added 'protein' and she didn't. But they're still similar." Not only was she keeping track of the ideas in the discussion, she was aware that similar ideas can be expressed in different terms. For Eleanor in that moment, the specific terminology was not as significant as the meaning the words conveyed. That contrasts with the 8th graders in lecture 2 at the outset of their thinking about the rock cycle, focusing on technical vocabulary they did not understand ("the Teutonic plates move and create rock"; "desa-whatever") . After their teacher pressed them to "start from what you know," they shifted to a sense more like Eleanor's, and their account became meaningful.

In the 3rd grade discussion about earthquakes, Skander explained his idea with an analogy: "Pretend the lava is water and the giant rock is a[n ice] cube." Not only was he invoking a familiar sense of physical mechanism, he was also choosing to make an analogy and finding that a useful thing to do. When his classmates made their criticisms, they focused on the specific mechanism the analogy conveyed, evidence they too understood the action of drawing an analogy as a way to convey an idea. That example contrasts with Louis's stance in lecture 2 at the outset of his physics course, when he did not see the use of analogies for thinking about electricity. After he failed an exam and the professor suggested he "try to explain it to a 10 year-old," he shifted in his stance.

Students' epistemologies are not as readily apparent as the content of their ideas or even their approaches. There is evidence, however, that they play a role in learning. Students at all ages have a variety of epistemological resources, and in assessing or planning an activity we can ask which aspects of their epistemologies they are likely to activate to understand what they are doing. That consideration has implications for instruction. For example, it calls into question the traditional attitude among physics instructors that students should not be expected to understand material the first time through. Such a pattern in students' experience, of ideas they do not understand, may lead to a counter-productive pattern in the epistemological resources they apply. The context of a physics course may come to activate resources for understanding knowledge as supplied by authority, residing in symbols and terminology, and disconnected from basic logic and common sense.

At the same time, from a view of manifold resources, an instructor would expect students are able to think of knowledge in other ways than they are at the moment. Diagnosing a difficulty as connected to a student's epistemology, an instructor could try to help the student find and activate more productive resources. [36]

2.4 Interpretive possibilities and instructional practices

Expectations of cognitive variability are embedded in much of expert instructional practice and curriculum development, as exemplified by lectures on pedagogy and curriculum in this volume. Viennot [40] identifies "critical details" in graphical representations of several concepts, subtle distinctions in depictions that can, evidently, tip student reasoning in one direction or another. Guidoni [41] suggests a variety of physical situations to pose as problems, each likely to evoke its own lines of reasoning; with the set challenging students to synthesize a coherent understanding. Heron's [42] account of curriculum development describes expectations of useful alternative knowledge in student reasoning and a process of helping students make use of it. There are other examples in my previous lectures, including Jessica Phelan's expectation that her 8th graders were capable of shifting their approach to the rock cycle and, perhaps more striking for her persistence, Trisha Kagey's expectation that her 3rd graders were capable of understanding the distinction between reasoning about mechanism as opposed to reasoning about purpose.

These practices all involve designing interventions and interpreting and responding to student reasoning with context in mind: Different presentations, problems, task framings and so on are likely to elicit different aspects of students' knowledge and reasoning. They go beyond, as Heron [42] made clear, the elicit-confront-resolve lesson structure of drawing out student conceptions, promoting dissatisfaction, and guiding toward alternatives, familiar from the literature in conceptual change [5] and based on attribution of stable conceptions.

So if expert practices already reflect tacit expectations of manifold resources, why bother with these theoretical questions about ontology? Ultimately, we would hope to achieve educational theory that (as theory has achieved in physics) provides a unifying structure to organize and advance our thinking about knowledge, learning, and instruction. We do not yet have one, and so we rely heavily on intuitive reasoning about minds. That reasoning, fortunately, is like intuitive physics in that it is adaptive and sensitive to context. By the same token it is often inconsistent, and for good or ill it includes ontological attributions. Theoretical work on ontology is largely the attempt to make those attributions explicit, examine them, and refine them toward technical precision. That is, in education research as in physics, we should look for progress in a "refinement of everyday thinking" [43]. It will come, we expect, through an exchange of insights between education research and instructional practice.

I shall have more to say about the role of theory below, in discussing the implications of different perspectives for objectives and assessment in elementary science education. Here I have highlighted ways in which a manifold view of resources is consonant with instructional expertise; the value in this respect is to describe aspects of expertise that have mostly remained tacit. In section 3, however, I discuss ways in which instructional practices reflecting unitary frameworks may have had deleterious effects on elementary science education. There, I argue, this shift in ontology is more than academic, even in the near term: It suggests the need for fundamental change in early science instruction.

2.5 Questions

At the end of the second lecture I listed some areas for research in phenomenology. Before I proceed to the final section of these lectures, I outline corresponding areas for research from a resources perspective.

1) What sorts of resources can we identify and describe?

We are only at the beginning of identifying and describing the sorts of resources students are likely to have available in their repertoires. Researchers often bemoan the number of different meanings for the terms such as "schema," "mental model," and now, perhaps, "p-prim" and "coordination class." It is daunting to reflect on how, for hundreds of years, physics struggled similarly over different meanings for terms to describe motion. If the comparison is apt, then we are likely as a field to remain at this beginning for some time. We may need to take a long-term view with respect to the practical value of this work, although the general shift in ontology has implications in the short term, as I have discussed above and will discuss further below.

2) How to model stabilities and shifts?

A unitary model of knowledge and reasoning accounts easily for stabilities, attributing them to structural units. Within a resources perspective, stabilities are emergent relationships among several structural units—the "coordination" of coordination classes [11,13]—and different kinds of stabilities need different kinds of models. Rosenberg, Hammer, and Phelan [44] present a toy-model analysis of the stabilities evident in the rock-cycle discussion, before and after Ms. Phelan's intervention, as small networks of resources. A strength of this approach is that it affords distributed accounts of cognition [45], as elements in stable networks can and generally must be distributed within and without individual minds [46,47]. In the case of the rock cycle, the presence and form of the worksheets played a role in the networks. Features of a particular problem [11,13,41,42], graphical representation [40,45], or social setting can be seen as parts of a stable network of activations.

Specific predictive studies have been unusual. Elby [48] showed that a particular fine-grained model of student reasoning could produce predictions with respect to student errors interpreting motion graphs: Graphs with certain prominent visual attributes would be more likely than others to trigger one set of resources for interpretation. Lising and Elby [34] designed their interviews of Jan with the expectation that they would preferentially activate different epistemological resources. Thagard's [16,49] model of explanatory coherence has similarly produced specific predictions regarding student reasoning in physics.

3) What existing research bears on this work?

As I noted, there have been a variety of manifold views of mind explored, in versions of schema theory and computational models. Siegler [8] reviews some of this work, but not to Black's [50] standards. Certainly I have not attempted a comprehensive review here. Redish [46,47] points to the relevance of linguistics research on "frames" as relevant to stabilities and to neuroscience for insights into mechanisms of activation and reinforcement. No doubt there is more to glean from research in early development, memory, metacognition, mathematics education, and discourse. As Black [50] argued, much of the challenge is in finding the relevant insights that may be available across a wide range of communities.

4) How to model development from childhood to professional scientists?

One of Siegler's [8] principal arguments for a manifold ontology is that it provides a mechanism for development, something missing from unitary accounts. He presents an analogy to biological evolution: Cognitive variability allows for adaptation of thought, as more successful patterns of thinking are used more often and, over time, reinforced. (Like Redish [46] he alludes to sympathetic findings from neuroscience research.)

Specifically with respect to physics education, we need research to identify productive resources as they appear in children's reasoning, the better to design appropriate instruction for children. In general, elementary science instruction has been designed around stage-based developmental theories that systematically underestimate the range and possibilities of children's reasoning [8,19]. Physics education researchers are in an advantageous position to contribute to this work: Researchers familiar with expert science may be able to recognize nascent forms of understanding and reasoning. Research in this vein would be of benefit as well to university instruction: Instructors who know what children can do may be in a different position to diagnose the difficulties and possibilities of their students' thinking.

I pursue this direction a bit further in the following section.

3 Elementary science education

Research in physics education, as in other fields, spans a spectrum from "basic" to "applied." At one end, there is foundational work that may not affect instructional practices for some time, if ever. Of course, basic research in any field has these same risks: It is easier to see in hindsight than in foresight which areas were worth patience and perseverance. At the other end of the spectrum is work with immediate motivation and connection to instructional practice. That work is also important, and clearly has an impact, but it comes with risks of its own: Applied research in the absence of a coherent theoretical foundation may involve flawed assumptions that steer instruction in wrong directions.

Current practices in elementary science education are largely motivated by unitary perspectives of conceptions, stages of development, and process skills. Content objectives and methods are crafted on the view that instruction should address student misconceptions, in the sense that it will promote change to new conceptions. Process objectives are shaped by the view that instruction should promote skills of science accessible to children given their stages of development, such as observing and measuring for younger children, with more abstract stills of theoretical argumentation deferred to later grades.

Observations of variability in student knowledge and reasoning challenge these perspectives. We do see coherences, but they are local, and the theme that emerges is that students are capable of multiple coherences in their understanding and reasoning. I have also reviewed theoretical arguments that attributions of unitary conceptions or stages of development do not account for or provide a basis from which to analyze the contextual variation we see in students' knowledge and reasoning. Along the way I have called attention to student epistemologies: Evidence of local coherence suggests they play a role in science learning, and evidence of multiple coherences suggests manifold epistemological resources.

Detailed study of the nature of these various resources is at the basic end of the spectrum, and the practical rewards of fine-grained modeling may not result for some time. But the general features of a resources-based perspective have immediate implications, including to suggest that teaching practices founded on unitary views can be deleterious. Instruction designed to elicit and confront misconceptions places too much emphasis too soon on the correctness of student ideas; instruction designed by stages of development significantly underestimates children's abilities. In this closing section, I illustrate these arguments using the example of the third-grade discussion about earthquakes from the first lecture and, finally, suggest a new direction for thinking about elementary science education.

3.1 Different perspectives on a class discussion

The examples of children's inquiry in the first lecture have elicited strong and divergent reactions. The third-grade discussion about earthquakes is an excellent case in point. It centered on a child, Skander, who ventured the idea that an earthquake could be caused by a large rock falling into lava under the earth's surface: The rock falling would make the lava rise and press against the ground from underneath. He explained his idea with an analogy to an ice cube dropping in water. When other students challenged that analogy by saying the rock would melt, Skander responded that the rock melting would make more lava, which would also take up space and have the same effect. In section 2, I discussed this case as an example of children's generating and thinking about an analogy, of drawing on everyday experience, and of reasoning with a broadly useful sense of conservation.

For some educators, though, it is an example of children's misunderstanding earthquakes and the process of scientific inquiry. Their concerns focus on the (1) the correctness of the students' understanding of earthquakes and (2) the correctness of their understanding of scientific inquiry. The first concern is that Skander's explanation is incorrect. More to the point, it was founded on a common but mistaken notion about what happens in an earthquake, that the ground cracks open and lava comes out, and this notion went unchallenged. With respect to students' progress at understanding the ostensible content of the discussion, from this perspective things could not have been much worse. The second concern is that the students were not reasoning from evidence—they had made no observations of earthquakes. In this the conversation was cultivating a wrong impression for how science proceeds. The "tragedy" of the moment (as one educator described it) is all the more poignant given the students' evident engagement, because it could have been directed toward accessible material.

I argue that these concerns reflect unitary views. More specifically, the reaction that a discussion like this *should not occur* derives from unitary understandings of students' knowledge and beliefs.

Someone with a manifold view could see how a problem would arise if conversations such as this were the only or predominant way children experience science, but there is little risk of that.

The first concern is that children are already at risk of hold the misconception that the ground cracks open in an earthquake. From that perspective, this discussion was destructive, because students could come away with the misconception only further engrained. From the view of manifold resources, the idea of the ground cracking open in an earthquake is not likely to be an element of cognitive structure. It could easily come up in discussions, partly as a result of cartoon depictions of earthquakes, but it is perfectly plausible that the same children, visiting a science museum with an earthquake simulator (or experiencing an actual earthquake) would never think to wonder why they do not see any cracks in the ground. That it is bandied about in this discussion would make little risk of it becoming theory-like, because theory-like ideas are difficult to achieve. Of course, we would not want the teacher telling students false information, but she was not doing that; in fact, much of the point for her was that the students understood the conversation to be about their thinking and not hers.

By the same token, I should note, from a resources view we should not expect that Skander or anyone else now "has" the concept of conservation. In fact, his original idea of a rock falling in lava violated conservation—the rock must have taken up volume before it fell, if the lava was 'full to the brim' as he later argued. Different questions in different moments activate different resources. The benefit of Skander's productive use of conservation reasoning, to argue that the melted rock still takes up space as lava, was incremental in adding experience with the idea and of how it can be useful. Siegler [8] describes that increment in terms of selective advantage over those other resources, by analogy to an instance of an animal with a trait surviving. The more such experiences he has, over his education, the greater the role conservation resources are likely to play in his reasoning.

Regarding the concerns about what the students were learning with respect to the nature of scientific inquiry, I must first respond that it would simply be in error to suppose that scientific inquiry always proceeds from established evidence. The history and sociology of science is replete with examples of theoretical inquiry preceding established empirical findings, based on assumptions no one has thought to check, on flawed or incomplete evidence, and even on speculations and guesses. Science does not in general proceed in an orderly manner from valid observations to theory, and we should at least be willing to pause and reflect on whether it is appropriate to assess classroom inquiry by standards professional science does not generally meet. Students would be right to conclude that science so depicted is something they could never do.

That said, the concern (or positive appraisal) that this activity could give students an 'impression' about the nature of science , again, reflects a unitary view that beliefs students apply or form in one situation represent what they do in general. On a resources view local impressions happen all the time, but they are local. The same students who seem to think it is not necessary to check on the known observational facts of earthquakes would, in another moment, be strongly inclined to 'go see what really happens.'

In other words, we should expect novices are quite flexible in their understandings, and this should temper our expectations for the good or harm any particular activity may do. The problems come from long term patterns in what they experience. If we are concerned that students come to a sophisticated understanding of the nature of science, we would certainly not want to design instruction systematically around reasoning in the absence of verifiable information. But neither would we want to design it so that students only experience clear, reliable findings pointing directly to the canonical ideas.

Unfortunately, science instruction is systematic in several respects, including in the way it values "correct" ideas over students' reasoning. Students are not so easily constrained, happily, and even after years of traditional science instruction, it is possible to enjoin them to a different kind of participation. Still, the pattern of what they experience is clear, and the older the students the more likely they are to think of science as divorced from common sense, comprised of correct information accessible only to authorities and geniuses. This is the possible harm of the misconceptions perspective, that it can heighten educators' pervasive concerns that students 'get it right.' There is reason to believe it has just this effect, such as in educators' concerns that allowing wrong ideas to go unchallenged, even in a 15 minute discussion among third-graders, is dangerous because it may create or support wrong conceptions.

In sum, from a unitary view of knowledge and reasoning, one should be concerned about students forming or having reinforced incorrect conceptions and beliefs about science, because these will be impediments to expertise. Whether with respect to concepts or reasoning, we want to be careful that students construct ideas and practices that are generally valid; we want above all to avoid their constructing ideas and practices that are not; and from a unitary view both of these are plausible if not likely outcomes from science instruction. In contrast, from a resources view, we expect variation. The ideas and practices students show or construct in one situation are not likely to apply in general. We should be less concerned about their wrong ideas, and less pleased over their correct ones, because over time they will be assembling, disassembling, and reassembling these ideas and others.

3.2 Promoting resources for tangible causality and coherence

This is certainly not to suggest that correctness is irrelevant. *Of course* at some point we want students to arrive at an understanding of established ideas. But *at what point* and *how* should we expect that to happen?

Current thinking about educational objectives in science are split between two sorts of consideration. One regards students' general interest, engagement, and motivation; the other their understanding. Instruction focused on the former is successful if the students are active, enthusiastic, and thinking. Content objectives are met if students make progress through the gradual, hierarchical assembly of correct conceptions. This dichotomy of concerns produces a dichotomy of experience in science. Children have fun with explorations in which any idea is good —pedagogy educators unfortunately often refer to as "constructivist"—and then, when concerns for understanding come into play, children are held accountable to the canon.

My purpose at the end of this chapter is to propose a third sort of consideration. I do not propose to eliminate objectives of interest and motivation or of canonical understanding. But 'anything goes' is not a productive attitude toward teaching or learning in science, and the evidence and arguments of variability cast doubt on views of progress exclusively in terms of the gradual assembly of correct conceptions. At some point we want to guide students to construct correct ideas, but on this view children's constructions in early grades are local acts of reasoning, not permanent conceptual change. Most of what they construct, whether it is correct or incorrect, they will be taking apart and putting back together many times over during their education. I propose we focus more on the taking apart and putting together.

There was not an objective in that third-grade lesson that students come to a correct understanding about earthquakes, nor was the objective simply their engagement and interest. It was neither hierarchical concept attainment nor anything goes. Rather, it was to draw out and support ways of thinking and talking that are, in general, productive for physical science. Thus the teacher challenged students to be clear about their thinking and internally consistent, to attend to each others' reasoning and pursue implications, and to favor mechanistic explanations over non-mechanistic ones.

The value, like the value I supposed of my son's thinking about why the poster fell from the wall in the anecdote I told in the first lecture, was in the students' use of resources that will be productive in many contexts of scientific reasoning. It was a bit of (1) practice with different pieces of mechanistic knowledge, here including the ideas of rocks pushing up lava and of matter (solid or melted) taking up space; (2) practice with a kind of activity, here including analogy generation and argumentation; and (3) opportunity for children to become aware of (1) and (2) as part of what it means to reason in science. The students were "messing about" [51] with ideas and ways of thinking—resources of various kinds—that will be useful in some (but not all) contexts they will encounter.

I am suggesting an approach to science education in terms of promoting productive resources, and while I am most concerned with a shift in how we view early science, this reframing could serve more generally to help us organize how we think about curriculum and instruction from early science through adulthood. The problem is how to characterize the resources that might be the seeds of professional science, a serviceable rough cut given what we know. One characterization might be resources for *tangible causality* and *coherence*.

By *tangible causality* here I mean resources to support an understanding of physical mechanism—diSessa's p-prims model basic elements of this knowledge [10,11], but I mean to include as well the refinements of these basic elements and larger structures students construct. I say *tangible* to emphasize the importance of these ideas forming from what is familiar to students, expecting that part of students' development entails extending and refining what is familiar. Resources for tangible causality would include an intuitive sense of blocking, for example, as well as the resources students construct later such as to understand electrostatic repulsion and attraction and, still later, to understand mass/energy "causing" spacetime curvature. That is, children enter school science with a very rich starter-set of resources for understanding physical cause and effect, and instruction should focus first on their learning to associate these resources with physical science and then on their refining and developing a more sophisticated set.

By *coherence* I mean resources for assessing and developing the connections and inconsistencies among ideas—the logico-mathematical schemas in Piagetian and neo-Piagetian accounts of early childhood as well as the refinements of these basic elements. Resources for coherence would include a child's sense of agreement or disagreement, for example, and of symmetry and fairness, as well as later abilities of, in particular, formal mathematics. Again, this is a view of children as beginning their formal education with a rich starter-set of resources for understanding coherence, which they learn to apply to reasoning in science, and progress involves refining and building on those resources.

I also include epistemological resources, from which students build an understanding of what knowledge and learning in science entail. In this respect as well, we should expect children begin with a rich starter set, including resources for understanding various origins of knowledge (e.g. authority, imagination, inference), activities and forms (e.g. arguments, analogies, guesses). Later refinements would include more developed understandings of hypotheses and principled frameworks, evidence, correlations and models.

One implication of this is in what makes a "good" activity in early science. On a unitary view, a good activity is one in which students are likely to construct the correct idea; an especially nice activity is one in which children will, with some reliability, arrive at a canonical conclusion. On a resources view, there are other kinds of activities that are valuable, including activities that are just the opposite: They raise *many* possibilities in student thinking, and that is their value, as opportunities for students to mess around with a variety of different resources in their repertoire, of different sorts of mechanism, of possible stances and approaches to take with respect to disagreements and inconsistencies, of strategies for argumentation, and so on. The risks of organizing science education only around inquiries tightly guided toward canonical answers is that children will not activate resources for contending with the more complicated situations that are more prevalent in science, of multiple possibilities leading to alternative conclusions.

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In mathematics, a 3-manifold is a space that locally looks like Euclidean 3-dimensional space. A 3-manifold can be thought of as a possible shape of the universe. Just as a sphere looks like a plane to a small enough observer, all 3-manifolds look like our universe does to a small enough observer. This is made more precise in the definition below. A topological space X is a 3-manifold if it is a second-countable Hausdorff space and if every point in X has a neighbourhood that is homeomorphic to The variability of student reasoning, lecture 3: manifold cognitive resources. Authors. D. Hammer.Â 1. Introduction. 2. From unitary to manifold. 3. Elementary science education. â†“ more. â†‘ less. $35.00 / â‚¬27.50 / Â£22.00 Add PDF to cart. Empirical investigations of learning and teaching, part I: Examining and interpreting student thinking. Authors. Paula R. L. Heron. These resources are context-independent, and relate to patterns that our cognitive apparatus readily recognize. What is the difference? â€¢ Conceptions may be technically â€˜incorrectâ€™. â€¢Â Chemistry has the special characteristic of being largely explained in term of entities conjectured at a different scale Hammer, D. (2004) The variability of student reasoning, Lecture 3: Manifold cognitive resources, Proceedings of the Enrico Fermi Summer School in Physics, Course CLVI, Italian Physical Society. Available at http://www.physics.umd.edu/perg/papers/papers-ee.htm. The topic â€¢ The particulate nature of matter is part of the staple science curriculum diet of secondary students throughout the world. â€¢