STUDENTS, LOGICAL THINKING AND TEACHING EFFICIENCY: A MOROCCAN CASE

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Abstract

This paper argues that instruction within the classroom generally helps students develop mathematical thinking skills. The research was conducted in a North African context, where students often face the challenge of thinking, learning and talking in different languages. The educational backgrounds of the students were diverse, with some students coming from a French educational background, while others came from schools that use Arabic as the medium of instruction. The paper includes a literature review in addition to a test that was administered to students before and after a semester of learning mathematics in class.

Key Words: College mathematics education, Logic, Teaching efficiency, Morocco

Introduction

The literature is rather united in terms of the current state of mathematics education, the causes for the current state of affairs, and the steps needed to overcome the problems it faces. The typology used to outline the literature focuses on the *state* of Mathematics Education, the *causes* identified in the literature as probable factors in the current problematic state of affairs, and the *solutions* suggested to address the problem. In its summation, this literature review will place our current efforts in the context of the literature. The literature on College-level Mathematics Assessment was sampled as part of our effort to evaluate our teaching process. Using standard academic journal databases, such as ERIC and JSTOR, we were able to extract a sample of the literature. As can be expected, the literature is dominated by quantitative approaches (Frise, 1996; Hoyt & Perera, 2001; Ratcliff & Yaeger, 1994; Stage & Kloosterman, 1995), although there were some papers that utilized qualitative approaches (Luk, 2005; Jordan and Haines, 2003; Cerrito, 1996). There were few controversies and disputes in the literature, suggesting that teaching mathematics has become, to some extent, a regular profession. Controversy, such as it was, is limited to the students' assessments of their instructors (Hoyt & Perera, 2001). In most other topics, there was widespread consensus concerning what ails mathematics instruction, not only in the United States, but also in Hong Kong and other societies. We were not able to find Arabic language sources on mathematics teaching evaluations, and we apologize to any colleagues in Morocco and the rest of the Arab world who may have conducted such research for failing to include their work.

The State of Mathematics Education

Mathematics education is not experiencing its best time. A 1995 study by Steve Bauman and Bill Martin at the University of Wisconsin at Madison, found that many students were incapable of reading a simple graph. They report that even engineering students were not able to identify some of the simple concepts they tested. Bauman and Martin also detail problems in knowledge retention in students who are not mathematics majors (Bauman & Martin, 1995). Their findings are reiterated by Hing Sun Luk, a professor of Mathematics at the Chinese University of Hong Kong, in an essay addressing the difficulties facing Mathematics education in Hong Kong. In his 2005 article, Luk argues that even elite students face significant problems learning and understanding mathematics. Luk locates the source of problems with Mathematics education in the heuristic gap between the methods used to teach mathematics in High Schools and Universities. The professor is also clear that much of the blame lies on "epsilon-delta gap between calculus and analysis." There is also some discussions of the "psychology" of learning mathematics, which is also a position taken by other authors (Luk, 2005). Like Bauman, Martin and Luk, Patria Cerrito argues that there is widespread ignorance of basic mathematics (1996). The extent of the problem is so widely acknowledged that a new term, QL (Quantitative Literacy) has appeared to describe the problem (Jordan & Haines, 2003). Quantitative illiteracy and other forms of innumeracy can hamper development efforts and stymie citizens in both the developed and the developing worlds in their efforts to make appropriate decisions as consumers, voters and workers.

The lack of preparation for and interest in Mathematics by the students is identified as a major obstacle for mathematics instruction by Drs. Chaatit and Marzouk in all three branches of Al Akhawayn University Ifrane. Using a sample drawn from the same population used in this study, the team finds that AUI students are generally not prepared to take classes, unwilling to apply themselves to the study of mathematics, and generally unable to perform properly in mathematics (Chaatit & Marzouk, 2007). In general, the problem of mathematics education is now identified as an education priority in the North (Sons, 1992). But as Luk's work suggests, the presence of the problem in Hong Kong suggests that this is a universal problem that is transnational and may not be limited to the older parts of the developed world and the global South.

The Causes of Quantitative Illiteracy

Luk's focus on psychological factors is upheld by studies using quantitative methods. One such study by Frances K. Stage and Peter Kloosterman, professors at the Indiana University at Bloomington, uses a sample of 236 students to test whether their beliefs about mathematics, their prior preparation and their gender influences their performance on remedial mathematics courses. For the male students, prior preparation had greater influence than it did for female students, but for both genders, the greatest predictors of mathematical performance were the student's beliefs about mathematics. Kloosterman and Stage define "mathematical beliefs" as 1) *confidence* in being able to solve difficult problems; 2) *steps* versus concepts in solving word problems; 3) *word problems* being an essential element of mathematics; 4) that *understanding* mathematics involves memorizing formulae; 5) that *effort* can improve one's mathematical performance (Stage & Kloosterman, 1995, pp. 296-297). The authors find that the student's mathematical belief set is the best predictor of their performance in the classroom.

Flowing along the same lines of reasoning, Donald Hoyt and Subashan Perera, researchers at the IDEA Center of Kansas State University, find that student motivation is the best predictor of their performance in quantitatively-oriented courses. Using a study of at least 1500 students, the authors gathered data from 50 classes from each of their "three levels of quantitative involvement: none, some and much" (Hoyt & Perera, 2001, p. 3). Interestingly, they also found that students tended to retaliate against social and behavioral sciences classes that had quantitative content by assigning their instructors lower levels of teaching effectiveness. The students' lack of motivation was working against them and ultimately also against their instructors. They defined motivation as a synthetic composite variable based on the students' responses to being asked to rate their response on a five-point response scale from "definitely false" to "definitely true." The students had to respond to the following statements:

I had a strong desire to take this course.

I really wanted to take this course from this instructor.

I really wanted to take this course regardless who taught it.

A fourth item was indirectly related: I worked harder on this course than on most courses I have taken. (Hoyt & Perera, 2001, p. 4)

Data from the College of the Canyons, a California Community College, suggest that students are often aware of their own shortcomings in Mathematics. In an August 1996 study, Dan Frise, correlates the students' self-assessment of their own mathematical skills with their scores on the intake test used by the College of the Canyons. The correlation appears strong enough to suggest that students may have a realistic picture of their ability, but that testing should proceed to safeguard their proper placement in classes that address their ability levels (Frise, 1996). This suggests that assessment testing is seen as one of the best methods with which to gain a handle on possible solutions to the problem of diminished QL.

Solutions to Quantitative Illiteracy

While the solutions are labeled with different titles and names, they do share one thing in common: the proper integration of mathematics into the wider curriculum without expecting the mathematics programs alone to correct for the problem of innumeracy. One of the largest studies in

our sample focused on measuring improvement among students and the coursework they pursued that helped them improve their mathematical performance. James Ratcliff and Patricia Yaeger, from the Pennsylvania State University's Center for the Study of Higher Education, conducted a study using data from a US institution of higher learning dubbed "Eastern College" – a pseudonym designed to protect the institution's privacy. They used transcripts and test scores from about one thousand students and examined about 900 courses. The focus of the study was about 100 students and 252 courses. This sub-sample included students with high verbal scores and low quantitative scores in the SAT and GRE standardized examinations. Ratcliff and Yaeger use the SAT as the baseline to include students in this special sub-population and then measure the students' performance in the GRE four years later. They were particularly interested in course sequences and clusters that improve the students' mathematical performance. The course cluster that showed gains in regular mathematics and quantitative comparison included courses in Mathematics, Accounting, Finance, Chemistry, Computer Science, as well as Political Science and Art History. Two other course clusters showed improvements in the students' mathematical skills, including one cluster based on psychology courses and another which was business oriented. Ratcliff and Yaeger also note that there was a cluster favored by students who wish to avoid all quantitative work. These students did not see any improvements in their mathematical skills during their four year college sojourn. Ratcliff and Yaeger conclude by suggesting that some forms of course work lead to improvements in the students' mathematical skills (Ratcliff & Yaeger, 1994).

While Ratcliff and Yaeger look at curriculum clusters, Patricia Cerrito, a professor of Mathematics at the University of Louisville, argues for the integration of mathematics and quantitative reasoning across the curriculum. She advocates an approach similar to the "writing across the curriculum" method used against poor English language skills among college students. In a short essay, she points out how mathematical concepts can be integrated into history and political sciences courses in a manner that teaches the students some basic concepts while retaining their political and historical relevance and content (Cerrito, 1996). Cerrito's positions are supported in an advocacy essay by Joy Jordan and Beth Haines of Lawrence University. Quoting an earlier work by Reed and Evans (1987), Haines and Jordan argue that mathematics needs to return to using the heuristics of analogy. They find that statistics is particularly helpful in improving student's quantitative skills. They also advocate the policies adopted by four institutions of Higher Learning in the United States that seek to integrate quantitative methods into the traditionally less quantitative fields like art, public policy and the social sciences (Jordan & Haines, 2003).

The sole apparent deviation from standard lines in the literature sample concerning solutions comes from Luk, who emphasizes the most "traditional" and at the same time "radical" approaches to mathematics. Perhaps his teaching philosophy can be best described as a "get tough but real" approach. He calls for bringing back the classics like Euclid and Hilbert. He calls for faculty to accept students as they are: clumsy, confused, ignorant and sometimes annoying. Luk challenges mathematics instructors to begin talking to the students in a manner they understand and warns against attempting to select the "best" of the students at the expense of average and weak students.

> "While we hear publicity about attracting and accepting the best students, we also hear ready laments about the less able students. What dissonance! It may be that our own mission lies in starting from where the students are and leading them as far as they can go. Who knows at the school-university transition how far that will be for any one for them?" (Luk, 2005, p. 172).

If one examines Luk's arguments and compares them to the consensus mathematics- within-a-curriculum arguments, it is easy to see that the two approaches are complimentary. Instructors may need to return to the basics and spend the time and the energy needed to guide the weaker students. The curriculum may be the factor that enables them to apply the abstractions of mathematics into the concrete realities they study within their majors.

Our Study and the Literature

Our current efforts are an attempt to find out how our students are performing in some basic mathematical tasks. There are other studies that did so also, such as the effort by Bauman and Martin during the 1990s. There are some differences in extent, sample size and the testing instruments. There are issues that must be addressed in the next phase of studies: demographics, gender, motivation, beliefs about mathematics, and prior preparation. Not all of these issues are currently

addressed in our sample, but it may be possible to duplicate, in miniature, the Ratcliff and Yaeger study, by using the students' quantitative results from their admissions test and then administering a second quantitative exam before the students graduate in the future.

As it stands, however, we are seeking to find out factors like retention, memorization versus learning, and teaching effectiveness. It is clear that these issues continue to haunt mathematics not only in the Arab world but also in the United States and the Far East. The focus, we believe, should be on the effectiveness of mathematical instruction. The primary measure for this would be whether and how students apply mathematical thinking to very basic problems. To that end, we turned to work that focuses on the absorption of logical patterns of thought by students; we found that an old 1973 contribution by Thomas C. O'Brien, then professor of Mathematics at Southern Illinois University, came closest to measuring thinking and logical patterns and whether students become more mathematical or logical in their thinking as a result of instruction. O'Brien's approach offered us several advantages. First, it evaluates the effectiveness of the instruction rather than the prior preparation of the students. Second, the approach allows us to test for the skills that students should carry with them from the mathematics classroom to the rest of life. Third, the method is practical and easier to apply than the alternatives discussed earlier. To update it, we added a battery of standard demographic variables, and given linguistic diversity in Morocco, we included language variables.

Methodology

A paper and pencil test was filled out by the students both at the start and at the end of the semester. The students' name was replaced by a letter code in both tests, to enable comparison between first and second administration of the test and to guarantee anonymity for the student. The first test contained two parts: demographics and language, and logic assessment. The second test only contained the logic assessment. The demographics and language component consisted of 11 questions about the students' social background (including parents' level of education), as well as about age and gender, and languages spoken and used in class. More specifically, the following questions were asked:

1. Age: _____
2. Gender: _____ (Male or Female)
3. Language(s) Spoken at home:_____
4. Baccalaureate type (Science, Literature, Economics etc.):_____
5. High School Attended with type (Moroccan Public, Moroccan Private, French, European, or American pattern): _____
6. Language used to discuss mathematics with classmates: _____
7. Language used to think about mathematics: _____
8. Language used to take notes in this class: _____ _____
9. Father's educational attainment, if known:
   a. Less than High School
   b. High School degree (Bacc.)
   c. Bachelor's degree (license)
   d. Master's degree or doctorate
10. Mother's educational attainment, if known:
    e. Less than High School
    f. High School degree (Bacc.)
    g. Bachelor's degree (license)
    h. Master's degree or doctorate
11. In general, I consider my family to be:
    a. Poor   b. Working Class   c. Middle Class   d. Well-off

The logic assessment part consisted of 48 questions as in O'Brien's work. While O'Brien had two instruments one causal in context and another class inclusion in context, we choose to have only one type of instrument because of the size of our sample. The type of instrument chosen was class

inclusion. The items in the test were based on the following three sentences:

1. If the scorpion is black, then it is fast.

2. If the cat is small, then it is grey.

3. If the eggplant is big, then it is bitter.

These three sentences were given in each of four inference patterns (modus ponens MP, contrapositive CP, inverse IV and converse CV) and in each of the four modes $p \Rightarrow q$, $\overline{p} \Rightarrow q$, $p \Rightarrow \overline{q}$, and $\overline{p} \Rightarrow \overline{q}$. So the logic test consisted of 48 items in total, which were given in random order according to each of the sentences. Two of the 26 initial students did not return for the post-test.

The test was administered during class. Students were given 40 minutes. Before the test, there was an introduction about the purpose of the test (including explanation about the demographics questions and about the meaning of the answers, in particular not enough clues). Anonymous treatment of the answers and results was stressed, as well as the fact that the results on the assessment would have no impact on the class grade. The sample consisted of 24 students in MTH1304 Discrete Mathematics for Engineers in Spring 2008. Students repeating the course or students who withdrew from the class in the course of the semester, were not included in the sample. MTH 1304 is an introduction to the fundamental ideas of discrete mathematics. Topics covered in this course include sets, logic, relations, recurrence relations, trees and graphs, partially ordered sets, lattices, Boolean algebras, algebraic structures. The aim of the course is to give the students a foundation for the development of more advanced mathematical concepts that are useful to computer science and computer engineering. Students are expected to achieve a balance among the computational skills, theory and applications of discrete mathematics.

This course has no pre-requisites. Most students taking it are freshman. MTH1304 is a required course for students majoring in Computer Science (CS) or in General Engineering (GE), and can be taken as elective by students majoring in Engineering and Management Science (EMS). As most students taking this class are freshman and as quite a lot of students switch between the majors CS, GE and EMS afterwards, it was decided not to consider the students' major as a variable.

The book used in MTH1304 was Discrete Mathematical Structures, B. Kolman, R.C. Busby and S.C. Ross, 5th edition. The following chapters were covered: Fundamentals, Logic, Counting, Relations and digraphs, Functions, Order relations and structures, trees, topics in graph theory. The chapter on logic is introductory. One section of this chapter deals with conditional statements or implications, including the contrapositive ~q => ~p of an implication p => q. The fact that an implication and its contrapositive are logically equivalent is established and used throughout the book. The class has 3 credits and meets three hours a week. Tutoring by an undergraduate student was available throughout the semester. Since attendance at the tutorial sessions was not required and very low, it was not considered as a variable.

Findings

Taking a class appears to improve the overall performance of students. As shown by Tables 1 and 2 below.   The sole exception appears to be converse pattern of inference in the $p \Rightarrow q$ mode, where the average score went down by 0.1.. These results are comparable to those found in the work being updated.

Table 1: Percentage Responding Correctly According to Mathematical Logic.

	Pretest					Posttest				
	$p \Rightarrow q$	$\overline{p} \Rightarrow q$	$p \Rightarrow \overline{q}$	$\overline{p} \Rightarrow \overline{q}$	Comp.	$p \Rightarrow q$	$\overline{p} \Rightarrow q$	$p \Rightarrow \overline{q}$	$\overline{p} \Rightarrow \overline{q}$	Comp.
MP	87.2	85	84.7	78.2	83.8	87.2	87.2	88.5	83.3	86.6
CP	56.4	32	61.5	41	47.7	61.5	44.8	65.1	53.8	56.3
IV	41	33.4	42.3	34.6	37.8	56.4	46.1	51.2	47.4	50.3
CV	52.6	39.7	50	44.9	46.8	52.5	46.4	52.5	48.7	50.0
Comp.	59.3	47.1	59.3	48.9	54.0	61.8	55.4	64.2	57.7	60.8

Table 2: Percentage Improvement

	Improvement				
	$p \Rightarrow q$	$\overline{p} \Rightarrow q$	$p \Rightarrow \overline{q}$	$\overline{p} \Rightarrow \overline{q}$	Comp.
MP	0	2.2	3.8	5.1	2.8
CP	5.1	12.8	3.6	12.8	8.6
IV	15.4	12.7	8.9	12.8	12.5
CV	-0.1	6.7	2.5	3.8	3.2
Comp.	2.5	8.3	4.9	8.8	6.8

We note that the largest improvement was in the patterns IV and CP, and in the modes $\overline{p} \Rightarrow \overline{q}$ and $\overline{p} \Rightarrow q$. When we look at the individual scores, we see that 77% of the students improved their average grade between pre- and posttest. We also note that, in comparison with MP, CP scores were low. We also notice, that IV is easier than CV. However, this difference seems to disappear in the posttest. This seems to point in the direction of other uses of logic. One of the conclusions of O'Brien is that consistent use of 'Child's Logic' persists in college students. We wanted to find out if this is also the case for our students. Since we have both pre- and posttest results for each student, we were also able to investigate whether there are any changes in interpretations of $p \Rightarrow q$ used.

For the terms 'consistent use' of an interpretation of $p \Rightarrow q$ and 'Child's Logic', we use the same definitions as O'Brien. A student is said to make consistent use of an interpretation in an inference pattern if 8 or more of the 12 items on that inference pattern are answered according to that interpretation. The term 'Child's Logic' refers to the interpretation of $p \Rightarrow q$ as $p \Leftrightarrow q$; in this case, $p \Rightarrow q$ is understood as p and q are either both false or both true. We indicate the correct mathematical interpretation of $p \Rightarrow q$ as 'Math Logic'. Finally, we also look at consistent use of NEC (not enough clues) as an answer for CP Table 3 below summarizes the percentages of students who make consistent use of one of the interpretations 'Math Logic', 'Child's Logic' or NEC for the different inference patterns MP, CP, CV, IV.

Table 3: Percentage Using Consistent (8 out 12) Patterns of Inference Getting the Correct Answer.

	MP	CP	CV	IV
Pretest				
Math Logic	87.5	20.8	29.2	45.8
Child's Logic	NA	NA	37.5	29.2
NEC	0	16.7	NA	NA
Posttest				
Math Logic	91.7	45.8	50	45.8
Child's Logic	NA	NA	29.2	29.2
NEC	0	12.5	NA	NA

We note an improvement in the consistent use of 'Math Logic' in CP and CV, while there is no improvement for IV. The percentage of students making consistent use of 'Child's Logic' in IV also remains the same.

We also looked at how many students make overall consistent use of one of the interpretations 'Math Logic', 'Child's Logic' or NEC; i.e. consistent use in each of the inference patterns MP, CP, CV and IV. In contrast to O'Brien, consistent use of 'Child's Logic' does not occur that frequently: only 8.3% of the students make consistent use of 'Child's Logic', in pretest as well as in posttest. In fact the same students who consistently use 'Child's Logic' in pretest, still use 'Child's Logic' consistently in posttest. We do notice an increase in the consistent use of 'Math Logic': while in the pretest only 8.3% of the students make consistent use of 'Math Logic', the percentage of students consistently using 'Math Logic' in the posttest is 20.8.

This still leaves a high number of students (83% in pre- and 71% in posttest) whose logical thinking patterns are yet to be revealed. Therefore we checked for some other interpretations of $p \Rightarrow q$ that have also been investigated by O'Brien. "1537," where students use "Math Logic" for MP and IV, NEC for CP and "Child's Logic" for CV. "1237," where students use "Math Logic" for MP, CP and IV, and "Child's Logic" for CV. "1534," where students use "Math Logic" for MP, CV and IV, and NEC for MP. "1564," where students use "Math Logic" for MP and CV, NEC for MP and "Child's Logic" for IV.

Table 4: Percentage Using a Key Consistently.

Key	Pretest	Posttest
Math Logic	8.3%	20.8%
Child's Logic	8.3%	8.3%
1537	4.2%	0%
1237	4.2%	4.2%
1534	4.2%	8.3%
1564	8.3%	4.2%

Demographic Factors

There were three age groups in the sample, 18, 19, and 20 year olds. The oldest category included three students whose performance improved from the pretest to the posttest. The 19 year olds improved least, with only 6 out of 10 improving their scores. The 18 year old students did better with 9 out of 10 improving their overall performance. One student choose not to mark his or her age. Consequently, it is likely that age is not a significant factor in the performance of the students. With regard to gender, the sample was nearly evenly split between men and women. Among the males 11 out 12 improved their performance. In addition 8 out 12 women improved their performance. There may be some differences due to approaches to studying between the two genders. The difference could possibly be related to initial approaches to the course. Classroom experience suggests that more of the women students approach studies more seriously initially, so the room for improvement may be narrower as a result. The variation in numbers suggests that there is a need for further study of this difference.

In terms of languages spoken at home, which is an important issue for an English-speaking university in non-English speaking country, 11 out of the 13 students whose households use Arabic exclusively registered an improvement in their performance; two of these students saw their performance regress. The remaining 11 students live in families that use Amazight, English, or French either solely or in combination with each other or with Arabic. Eight of the non-solely Arabic speaking students saw their performance improve. Two regressed and one student remained stagnant. All the students who stagnated or regressed were raised in multiple language households. Concerning the language used to think about mathematics, those using Arabic to think, improved their performance in 4 out of 7 cases. Those using English to think about mathematics improved their performance in 3 out of 4 cases. Those using French in combination with Arabic saw their performance improve in 1 out of 2 cases. The sole student using French with English to think about mathematics saw his or her performance regress. The remaining students used French (6), English (1) or Amazight (1) or a combination to think about mathematics, and they all registered improvement. This suggests that the issue of linguistic diversity needs further study.

In terms of class, the students considered themselves to be middle class by an overwhelming margin (17 out of 26 initial students). Only one student considered his family to be poor. Three considered their families to be working class, and five claimed to be well off. Of the 17 middle class students, 13 improved their performance in the posttest. The single student of poor background improved his or her performance in the post-test as well. Among the three students who considered themselves to be of working class background, two improved their performance while one regressed. It was only among the five students who considered themselves well off that the pattern of overall improvement came into question; two of these students saw their performance regress while the remaining three improved their results. We believe that the difference between middle class, working class, and poor students in one hand and well-off students on the other suggests that there is a warrant for further study of how class and wealth affect the study of mathematics.

Conclusion

Different patterns of logic are used by students. Further research in logic interpretations used by students could help in improving the teaching of logic. Only a small fraction of the students used "Child's Logic" consistently. There is a significant improvement in the consistent use of "Math Logic" by the students. This study also strongly suggests that mathematics instruction leads to results at least in some modes and patterns of inference. To that end, we find Luk's call for working with the students as they are correct. While improvement will never be universal, attempts to spread mathematics across the curriculum while reducing the mathematics requirements in the common core from three semester courses, to two and then one in some schools will come at the expense of improvements in the critical thinking and mathematical logic skills of students. Should a mathematics-across-the-curriculum approach be combined with a vigorous application of teaching mathematical basics, the results are likely to be better. We find that the problems facing mathematics are global. It is not simply a matter of preparation, because other school systems are facing the same challenges despite having a very solid foundation in K-12 mathematics education.

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